ECE 341: Probability and Random Processes for Engineers, Spring 2012
Homework 6
Name:
Assigned: 02.15.2012
Due: 02.22.2012

Problem 1. A certain deep space transmitter uses on-off modulation of a laser to send a bit, with value either zero or one. If the bit is zero, the number of photons, X, arriving at the receiver has the Poisson distribution with mean $\lambda_0 = 2$; and if the bits is one, X has the Poisson distribution with mean $\lambda_1 = 6$. A decision rule is needed to decide, based on observation of X, whether the bit was a zero or a one.

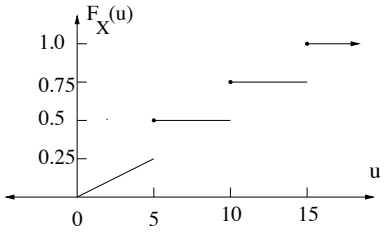
1. Suppose we decide that a 1 is sent if $P[X = k|\text{one is sent}] > P[X = k|\text{zero is sent}]$. What does this decision rule simplify to in terms of X?

2. Suppose we know that the probability of sending a zero is π_0 and that of sending a one is π_1. Suppose now that we decide a 1 is sent if $P[X = k, \text{one is sent}] > P[X = k, \text{zero is sent}]$. What does this decision rule simplify to now in terms of X if $\pi_0 = 5\pi_1$?

Solution 1:

Problem 2. Let X have CDF shown in Fig. . Find the numerical values of the following quantities:

1. $P[X \leq 1]$
2. $P[X \leq 10]$
3. $P[X \geq 10]$
4. $P[X = 10]$
5. $P[|X - 5| \leq 0.1]$



Solution 2:

Problem 3. Exponential random variables have a nice property termed the "memoryless property", which means that $P[T > s + t|T > s] = P[T > t]$, where T is exponentially distributed. Show this property.

Solution 3:

Problem 4. Suppose $Y = X^2$, where $X \sim \mathcal{N}(\mu, \sigma^2)$ with $\mu = 2, \sigma^2 = 3$. Find the pdf of Y. *(HINT:* $\frac{d}{dx}\Phi(x) = \frac{1}{\sqrt{2\pi}} \exp(-x^2/2)$*.)*

Solution 4: