

ECE 341: Probability and Random Processes for Engineers, Spring 2012

Homework 4

Name:

Assigned: 02.01.2012

Due: 02.08.2012

Problem 1. A Zipf $(n, \alpha = 1)$ random variable X has p.m.f.

$$P_X(x) = c(n)/x, \text{ for } x = 1, 2, 3, \dots, n$$

The constant $c(n)$ is set so that $\sum_{n=1}^n P_X(x) = 1$ (to make it a proper p.m.f.). Calculate $c(n)$ for $n = 1, 2, 3, 4, 5, 6$.

Solution 1:

Problem 2. The Zipf $(n, \alpha = 1)$ random variable X introduced in Problem 1 is often used to model the “popularity” of a collection of n objects. For example, a Web server can deliver one of n Web pages. The pages are numbered such that the page 1 is the most requested page, page 2 is the second most requested page and so on. If page k is requested, then $X = k$. To reduce external network traffic, an ISP gateway caches copies of the k most popular pages. Using Matlab, calculate, as a function of n for $1 \leq n \leq 100$, how large k must be to ensure that the cache can deliver a page with probability 0.75 or more.

Solution 2:

Problem 3. We measure for resistance R of each resistor in a production line and we accept only the units whose resistance is between 96 and 104 ohms. Find the percentage of the accepted units if

- R is uniform between 95 and 105 ohms
- R is Gaussian with mean 100 and standard deviation 2 ohms.

Solution 3:

Problem 4. The probability of heads of a random coin is a RV P uniform in the interval $(0, 1)$.

- Find the probability $P[0.3 \leq P \leq 0.7]$.
- The coin is tossed 10 times and heads shows 6 times. Given this fact, find the probability that P is between 0.3 and 0.7.

Solution 4: