ECE 341: Probability and Random Processes for Engineers, Spring 2012
Homework 1 - Solutions
Name:
Assigned: 01.11.2012
Due: 01.17.2012

Problem 1. If $A = \{2 \leq x \leq 5\}$ and $B = \{3 \leq x \leq 6\}$, find:

- $A \cup B$
- $A \cap B$
- $(A \cup B) \cap (A \cap B)^c$

Solution 1: Write solution here.

- $A \cup B = \{2 \leq x \leq 6\}$
- $A \cap B = \{3 \leq x \leq 5\}$
- $(A \cup B) \cap (A \cap B)^c = \{x = 2, 6\}$

Problem 2. We have two coins: coin A is fair and coin B has two heads. We pick one of the coins at random and toss it twice; heads show both times. Find the probability that the coin picked is fair.

Solution 2: Application of Bayes Rule. Name the event of having picked fair coin A; Event of having picked coin with two heads as B. Event of having two heads after two tosses as C.

$$P(A|C) = \frac{P(A,C)}{P(C)} \tag{1}$$

$$P(A,C) = P(A)P(C|A) = \frac{1}{2} * \frac{1}{2} * \frac{1}{2} = \frac{1}{8} \tag{2}$$

$$P(C) = P(A)P(C|A) + P(B)P(C|B) = \frac{1}{2} * \frac{1}{2} * \frac{1}{2} + \frac{1}{2} = \frac{5}{8} \tag{3}$$

$$P(A|C) = \frac{\frac{1}{8}}{\frac{5}{8}} = \frac{1}{5} \tag{4}$$

Problem 3. If $A \subset B$, and $P[A] = 1/4$ and $P[B] = 1/3$, find $P[A|B]$ and $P[B|A]$.

Solution 3:

$$P(A, B) = P(A) \tag{5}$$

$$P(A|B) = \frac{P(A, B)}{P(B)} = \frac{\frac{1}{4}}{\frac{1}{3}} = \frac{3}{4} \tag{6}$$

$$P(B|A) = \frac{P(A, B)}{P(A)} = \frac{\frac{1}{4}}{\frac{1}{4}} = 1 \tag{7}$$

Problem 4. Draw the probability table (matrix), and the tree diagram describing the following experiment: In a certain village, a random sampling of residents results in females having blue eyes with probability 1/3, females not having blue eyes with probability 1/4, males having blue eyes with probability 1/4 and males not having blue eyes with probability 1/6. Use the labels F_B, F_{NB}, M_B, M_{NB}.

Solution 4:

	Woman	Man
Blue	$\frac{1}{3}$	$\frac{1}{4}$
NotBlue	$\frac{1}{4}$	$\frac{1}{6}$

(8)

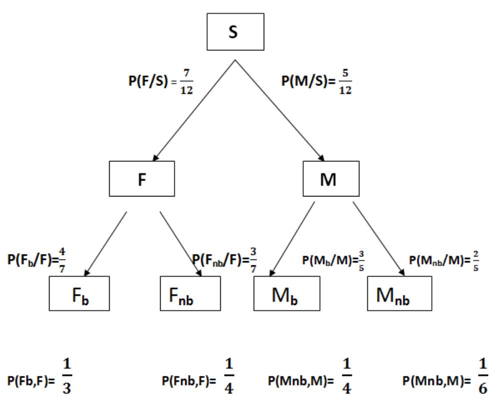

Figure 1: Tree diagram

$$\text{Given the } \textbf{Joint Probabilities} \tag{9}$$
$$P(F_b, F) = \frac{1}{3};\ \ P(F_{nb}, F) = \frac{1}{4} \tag{10}$$
$$P(M_b, M) = \frac{1}{4};\ \ P(M_{nb}, M) = \frac{1}{6} \tag{11}$$
$$\text{One can compute the } \textbf{Conditional Probabilities placed along the branches} \tag{12}$$
$$P(F) = P(F|S) = P(F_b, F) + P(F_{nb}, F) = \frac{1}{3} + \frac{1}{4} = \frac{7}{12} \tag{13}$$
$$P(M) = P(M|S) = P(M_b, M) + P(M_{nb}, M) = \frac{1}{4} + \frac{1}{6} = \frac{5}{12} \tag{14}$$
$$P(F_b|F) = \frac{P(F_b, F)}{P(F)} = \frac{\frac{1}{3}}{\frac{7}{12}} = \frac{4}{7} \tag{15}$$
$$P(F_{nb}|F) = \frac{P(F_{nb}, F)}{P(F)} = \frac{\frac{1}{4}}{\frac{7}{12}} = \frac{3}{7} \tag{16}$$
$$P(M_b|M) = \frac{P(M_b, M)}{P(M)} = \frac{\frac{1}{4}}{\frac{5}{12}} = \frac{3}{5} \tag{17}$$
$$P(M_{nb}|M) = \frac{P(M_{nb}, M)}{P(M)} = \frac{\frac{1}{6}}{\frac{5}{12}} = \frac{2}{5} \tag{18}$$
$$\tag{19}$$

Problem 5. If $A \cap B = \emptyset$, show that $P[A] \leq P[B^c]$.

Solution 5:

$$P(A \cap B) = 0 \tag{20}$$
$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = P(A) + P(B) \tag{21}$$
$$\text{But} P(A \cup B) \leq 1 = P(A) + P(B) \leq 1 => P(A) \leq 1 - P(B) => P(A) \leq P(B^c) \tag{22}$$