ECE 341: Probability and Random Processes for Engineers, Spring 2012
Homework 13 - Last homework
Name:
Assigned: 04.18.2012
Due: 04.25.2012

Problem 1. Let $X(t)$ be the input to a linear time-invariant filter. We are given that $X(t)$ is a wide sense stationary stochastic process with expected value $\mu_X = 4$ volts. The filter output $Y(t)$ is also a wide sense stationary stochastic process with expected value $\mu_Y = 1$ volt. The filter impulse response is $h(t) = e^{-t/a}$ for $t \geq 0$ and 0 otherwise. What is the value of the time constant a?

Solution 1:

Problem 2. A wide sense stationary process $X(t)$ with autocorrelation function $R_X(\tau)$ and power spectral density $S_X(f)$ is the input to a tapped delay line filter (this is a linear time invariant filter) with frequency response

$$H(f) = a_1 e^{-j2\pi f t_1} + a_2 e^{-j2\pi f t_2}$$

Find the output power spectral density $S_Y(f)$ and the output autocorrelation $R_Y(\tau)$.

Solution 2:

Problem 3. A wide sense stationary process $X(t)$ with autocorrelation function $R_X(\tau) = e^{-4\pi\tau^2}$ is the input to a filter with transfer function $H(f) = 1$ for $0 \leq |f| \leq 2$ (and 0 else). Find:

1. The average power of the input $X(t)$
2. The output power spectral density $S_Y(f)$.
3. The average power of the output $Y(t)$.

Solution 3:

Problem 4. Let X be a random variable (an i.id. source) with 7 outputs with probability mass function given by $[0.49, 0.26, 0.12, 0.04, 0.04, 0.03, 0.02]$ (the i-th element of this vector is the probability of the i-th symbol).

1. Find the entropy of this source.
2. Construct a Huffman code for this source.
3. What is the efficiency of the code you constructed in part 2?

Solution 4:

Problem 5. One is given 6 bottles of wine. It is known that precisely one bottle has gone bad (it tastes bad). From inspection of the bottles it is determined that the probability p_i that the i-th bottle is bad is given by $(p_1, p_2, \cdots, p_6) = (\frac{8}{23}, \frac{6}{23}, \frac{4}{23}, \frac{2}{23}, \frac{2}{23}, \frac{1}{23})$. Tasting will determine the bad wine.

Suppose you taste the wines one at a time. Choose the order of tasting to minimize the expected number of tasting requeired to determine the bad bottle. Remember, if the first 5 wines pass the test you don't have to taste the last.

1. What is the expected number of tastings required?

2. Which bottle should be tasted first?

Now you get smart. For the first sample, you mix some of the wines in a fresh glass and sample the mixture. You proceed, maxing and tasting, stopping when the bad bottle has been determined.

3. What is the minimum expected number of tastings required to determine the bad wine?

4. What mixture should be tasted first?

Solution 5: