ECE 341: Probability and Random Processes for Engineers, Spring 2012
Homework 12
Name:
Assigned: 04.11.2012
Due: 04.18.2012

Problem 1. We model the noon-time temperature in Singapore (in degrees celsius) as X_n on day n, where X_n is a sequence of i.i.d. Gaussian random variables with a mean of 30 degrees celsius and standard deviation 5 degrees.

1. Consider the new random process $Y_k = \frac{X_{2k-1}+X_{2k}}{2}$ (the average temperature over two days. Is Y_k an i.i.d. random sequence?
2. Consider another new random process $W_k = \frac{X_n+X_{n-1}}{2}$ (moving average of temperatures over two days). Is W_n an i.i.d. random sequence?

Solution 1:

Problem 2. If X_n is an i.i.d. random sequence with mean $E[X_n] = \mu_x$ and variance $Var[X_n] = \sigma_x^2$, what is the auto-covariance $C_x[m, k]$?

Solution 2:

Problem 3. Let $X(t)$ be a stationary process (strictly stationary). Consider $Y(t) = X(t + a)$; is $Y(t)$ also stationary or not?

Solution 3:

Problem 4. Similar to what we showed in class (but now you need to work it out), let $X(t)$ be a wide-sense stationary random process with average power equal to 1. Let Θ denote a random variable with uniform distribution over $[0, 2\pi]$, and let Θ and $X(t)$ be independent.

1. Find $E[X^2(t)]$.
2. Find $E[\cos(2\pi f_c t + \Theta]$ (show it, don't just quote class).
3. Let $Y(t) = X(t)\cos(2\pi f_c t + \Theta)$. What is $E[Y(t)]$?
4. What is the average power of $Y(t)$?

Solution 4: