ECE 341: Probability and Random Processes for Engineers, Spring 2012
Quiz 5, 04.18.2012
Name: Solution

X_n is a random sequence with mean $\mu[n] = E[X_n]$ and auto-correlation function $R_X[n,k]$ (we know both of these). We make noisy observations of this random sequence, i.e observe $Y_n = X_n + N_n$, where N_n is a random noise sequence with $\mu_N = E[N_n] = 0$, and autocorrelation $R_N[n,k]$. Assume that the noise sequence N_n is independent of X_n.

10 1. Find $E[Y_n]$.

20 2. Find the auto-correlation function of Y_n, $R_Y[n,k]$.

20 3. Find the auto-covariance function of Y_n, $C_Y[n,k]$.

Solution:

1. $E[Y_n] = E[X_n + N_n] = E[X_n] + E[N_n] = \mu[n] + 0 = \mu[n].$

2.
$$\begin{aligned}
R_Y[n,k] &= E[Y_n Y_{n+k}] = E[(X_n + N_n)(X_{n+k} + N_{n+k})] \\
&= E[X_n X_{n+k}] + E[X_n N_{n+k}] + E[N_n X_{n+k}] + E[N_n N_{n+k}] \\
&= \underbrace{R_X[n,k]}_{\text{definition}} + \underbrace{E[X_n]E[N_{n+k}]}_{\text{independent}} + E[N_n]E[X_{n+k}] + R_N[n,k] \\
&= R_X[n,k] + 0 + 0 + R_N[n,k].
\end{aligned}$$

3.
$$\begin{aligned}
C_Y[n,k] &= E[(Y_n - E[Y_n])(Y_{n+k} - E[Y_{n+k}])] \\
&= E[Y_n Y_{n+k}] - E[Y_n E[Y_{n+k}]] - E[Y_{n+k} E[Y_n]] + E[E[Y_n]E[Y_{n+k}]] \\
&= R_Y[n,k] - \mu[n+k]E[Y_n] - \mu[n]E[Y_{n+k}] + \mu[n]\mu[n+k] \\
&= R_Y[n,k] - \mu[n+k]\mu[n] - \cancel{\mu[n]\mu[n+k]} + \cancel{\mu[n]\mu[n+k]} \\
&= R_Y[n,k] - \mu[n+k]\mu[n].
\end{aligned}$$