ECE 341: Probability and Random Processes for Engineers, Spring 2012
Quiz 4, 04.02.2012
Name: Solutions

A man and a woman decide to meet at a certain location. Each person arrives independently, and at a time uniformly (continuous) distributed between noon and 1pm. Let X be the RV denoting the arrival time (between 0 and 60 minutes) of the woman, and let Y be the RV denoting the arrival time (between 0 and 60 minutes) of the man. Find:

1. The joint pdf $f_{X,Y}(x, y)$.
2. The marginal pdfs $f_X(x)$ and $f_Y(y)$.
3. Find the probability that the first person to arrive has to wait longer than 10 minutes for the second person to arrive.

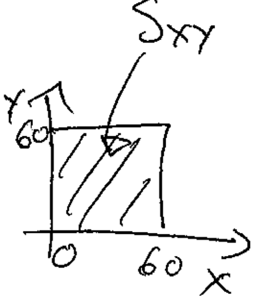
Solution:

1. Since X, Y are independent $f_{XY}(x,y) = f_X(x) f_Y(y)$.

Since X is uniform on $[0,60]$ $\Rightarrow$ $f_X(x) = \begin{cases} \frac{1}{60} & x \in [0,60] \\ 0 & \text{else.} \end{cases}$

Since Y is uniform on $[0,60]$ $\Rightarrow$ $f_Y(y) = \begin{cases} \frac{1}{60} & y \in [0,60] \\ 0 & \text{else} \end{cases}$

So $f_{XY}(x,y) = \begin{cases} \frac{1}{3600} & \text{for } (x,y) \in [0,60] \times [0,60] \\ 0 & \text{else} \end{cases}$



2. Solved in part 1.

3. We want $P[X+10 < Y] + P[Y+10 < X] \overset{\text{by symmetry}}{=} 2P[X+10<Y]$

$$= 2 \iint_{x+10 \le y} f_{XY}(x,y)\,dx\,dy = 2 \int_{y=10}^{60} \int_{x=0}^{y-10} \left(\frac{1}{60}\right)^2 dx\,dy = \frac{2}{(60)^2} \int_{10}^{60} (y-10)\,dy$$

$$= \frac{2}{(60)^2} \left[\frac{y^2}{2} - 10y\right]_{10}^{60} = \frac{2}{(60)^2} \left[\frac{(60)^2}{2} - 60(10) - \frac{(10)^2}{2} + (10)^2\right] =$$

$$= \frac{25}{36}$$