Let's Share CommRad: Effect of Radar Interference on an Uncoded Data Communication System

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***Abstract*—Spectrum sharing between radar and communications systems is currently under investigation because of the high demand for new wireless services and shortage of available bandwidth. To effectively design their coexistence, it is crucial to understand how current unaltered radar and communication systems would affect one another. This paper investigates the effect of radar interference on an uncoded data communication system where the optimal Maximum-A-Posteriori decoder is used and where the bandwidth of the radar system is much larger than the one of the communication system. Conclusions depend on how the radar interference power, measured by the Interference-to-Noise ratio (INR), compares with the intended signal power, measured by the Signal-to-Noise ratio (SNR). For the case of real-valued modulation schemes, three regimes emerge: (a) Treat interference as Gaussian noise: when INR<SNR it is optimal to use the threshold decoder for Gaussian noise only. The probability of error increases with INR. (b) Interference cancellation: when INR≫SNR the optimal receiver estimates the radar interference and subtracts it from the received signal; in the process of canceling interference, part of the useful signal is also cancelled, which reduces the effective SNR at the receiver. The probability of error exhibits an irreducible error floor, which can be *exactly* characterized and behaves like a narrow-band fading channel with multiplicative fading that is perfectly known at the receiver. (c) When INR≈SNR, the probability of error attains its maximum value, thus indicating that there is a worst operating INR for any given SNR.**

I. Introduction

Due to limited spectrum resources and increasing demand for wireless communications deployment, DARPA (Defense Advanced Research Projects Agency) has launched the SSPARC (Shared Spectrum Access for Radar and Communications) program to encourage research in this direction [1]. The NSF (National Science Foundation) also has a dedicated crosscutting program for Enhancing Access to the Radio Spectrum [2], [3]. The spectrum of interest for sharing is the S-band (2-4 GHz), in which several radar systems (i.e., air surveillance and weather) and wireless communication systems (i.e., Wi-Fi and WLAN) operate. The economical implications of successful spectrum sharing, as well as the technical challenges of maintaining integrity under spectrum sharing for each individual system, are difficult to understate.

A. Past Work

The effects of interference between radar and communications systems were already considered in the 50s [4], but have only recently really come to the forefront because of the many-fold increase of wireless data traffic due to smartphones, tablets and real-time video streaming. The longer term goal may be to modify radar and communications systems to co-exist (see [5] and references therein). However, to effectively design co-existence systems, it is also important to first understand the impact of unaltered systems on one another. In this vein, there are two important avenues to visit: how communications signals affect the performance of radar systems [6]–[12], and the reverse: how radar signals affect the bit error rate (and other metrics) of communications signals. This paper is interested in analytically characterizing the latter; we hence expand only upon the prior work in this domain, of which there is relatively little, to the best of our knowledge.

In [13], the authors simulate the effect of a rotating radar on a WiMax receiver, demonstrating the effect on bit and packet error rates, and [14] qualitatively proposes a model for how radar receivers may saturate the receivers of other services. In [15], coexistence issues for WiMax and radar systems in the S-band are again discussed qualitatively. In [16] the feasibility of dynamic spectrum sharing between air traffic control radar and wireless communications systems is investigated via a numerical simulation. In [17] spectrum sharing between multiple-input multiple-output (MIMO) radar system and a communication system modeled as MIMO interference channel is considered, where a zero-forcing precoder for radar transmitter which completely eliminates the radar interference to communication users is proposed and the impact on both the radar and communication systems is investigated.

While Bliss et al. venture into the theoretical analysis which looks at a tradeoff between communications information rate and a novel radar estimation rate [18], [19], we are not aware of an *analytical model available to justify and predict the effect of unaltered radar interference on the bit error rate for communication systems.* This is what we pursue.

B. Contributions

Our approach to understanding the co-existence between radar and communications systems is to first derive and analyze the optimal performance of a communication system in the presence of an unaltered radar system. We are not aware of any other works which attempt such an analytical characterization.

To do so, we need a tractable model. Radar signals typically consist of periodic pulses of large amplitude and short dura-

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tion, while communication signals usually have significantly lower power levels, 100% duty cycle and much smaller bandwidth. This implies that each narrowband subcarrier in an OFDM-based communication system experiences the radar interference as an approximately amplitude-constant additive interference. This amplitude can be accurately estimated from the knowledge of the (slowly varying) parameters of the radar waveform and the relative geometry between the radar transmitter and the communication receiver. The phase however changes very rapidly due to multi-path propagation; the worst case scenario is when such a random phase is uniformly distributed. With these considerations in mind, we build a simple model for when a radar and a communications system operate in the same frequency band, which captures some key performance bottlenecks and highlights different operating regimes. Extensions are discussed in the conclusion.

The major contributions of this work are as follows. For an uncoded BPSK (Binary Phase Shift Keying) communication scheme with AWGN (Additive White Gaussian Noise) and additive interference, with known constant amplitude and unknown random phase uniformly distributed in $[0, 2\pi]$, we analyze the BER (Bit Error Rate) performance for an optimal MAP (Maximum-a-Posteriori) decoder. We identify three different regimes of operation depending on the relative value of the radar interference power, measured by the INR (Interference to Noise Ratio), and the intended signal power, measured SNR (Signal to Noise Ratio). They are:

1) *Treat interference as Gaussian noise*: when INR $\leq$ SNR we show that it is optimal to use the BPSK threshold decoder as if the interference were Gaussian noise. This is optimal when INR $= 0$ and intuitively it should be optimal for small INR. What is interesting here is that the 'small INR' regime actually encompasses the whole interval INR $\in [0, \mathrm{SNR}]$—at least for sufficiently large INR and SNR, but it is actually slightly larger for small SNR. As expected, the BER increases with INR and this increase can be *exactly* characterized.
2) *Interference cancellation*: when INR $\gg$ SNR we show that it is optimal at the communication receiver to estimate the phase of the radar interference (as the phase of the received signal) and subtract its contribution from the received signal. Intuition would suggest that when INR is huge compared to all other signals in the systems, its contribution could be perfectly cancelled and thus the BER performance would be as if again INR $= 0$ (in line with the capacity achieving scheme for the two-user interference channel in the very strong interference regime [20]). What is interesting here is that, in the process of canceling the radar interference, part of the useful signal is also cancelled. This in turn reduces the effective SNR at the communication receiver and thus the BER exhibits an *irreducible error floor* in the limit for INR $\to \infty$. This error floor is *exactly* characterized here and it effectively behaves as if the desired signal is affected by a multiplicative/narrow-band fading known perfectly at the receiver.
3) *Intermediate regime*: when INR $\approxeq$ SNR, the BER attains its maximum value, thus indicating that there is a worst operating INR for any given SNR.

In a nut-shell, what the analysis points out is that the effect of a wide-band additive radar interference, of much larger power than the communication data signal, under optimal MAP receiver is the same as that of a multiplicative/narrow-band fading known perfectly at the receiver. This provides a nice model for spectrum sharing that can benefit for the large body of work already done for narrow-band fading channels with fading known perfectly at the receiver [21]. The difference here, compared to standard fading models, is that the multiplicative interference is distributed as the cosine square of a uniformly distributed random variable. This type of fading distribution has not been extensively studied except for the capacity of the "phase-noise non-coherent" Gaussian fading channel, for which the capacity achieving distribution is known to be discrete with countably infinite mass points [22], and whose asymptotic capacity when SNR is very large scales as $1/2 \cdot \log(1 + \mathsf{S})$ as opposed to the coherent case capacity that scales as $\log(1 + \mathsf{S})$ [23].

The paper is organized as follows. The system model is presented in Section II. The optimal BER for a BPSK MAP decoder is discussed in Section III. Section IV concludes the paper.

II. System Model

As per the discussion in Section I, the effect of a short duty-cycle radar pulse on a narrowband data communication system can be modeled as follows. At the communication receiver, the discrete-time complex-valued received signal is

$$Y = \sqrt{\mathsf{S}}X + \sqrt{\mathsf{I}}e^{j\Theta} + Z, \tag{1}$$

where X is the transmitted symbol from the constellation $\mathcal{X} = \{x_1, ..., x_N\}$ of unit energy and equally likely points, Θ is the random phase of the radar interference uniformly distributed in $[0, 2\pi]$, and Z is a zero-mean unit-variance proper-complex Gaussian noise. The random variables (X, Θ, Z) are independent. For the (without loss of generality) normalizations used in this paper, S is average SNR at the communication receiver, while I is the average INR. In the following we assume that the pair (S, I) is known at the receiver and fixed.

Our goal is to understand the average probability of error $\Pr[X \neq \hat{X}]$, where $\hat{X}$ is the estimate at the communication receiver of the transmit signal X, for the AWGN with additive radar interference in (1).

III. Optimal MAP Decoder and Error Rate Analysis

Let the channel conditional distribution be indicated as

$$f_{Y|X,\Theta} := \frac{1}{\pi} e^{-|Y - \sqrt{\mathsf{S}}X - \sqrt{\mathsf{I}}e^{j\Theta}|^2}.$$

The optimal MAP receiver, when the received signal is $Y = y$, chooses as estimate of the transmit constellation point

$$\hat{\ell}(y) = \arg \max_{\ell \in [1:N]} \Pr[X = x_\ell | Y = y]$$

$$= \arg\max_{\ell\in[1:N]} \mathbb{E}_\Theta[f_{Y|X,\Theta}(y|x_\ell,\Theta)]$$
$$= \arg\min_{\ell\in[1:N]} \Big(|y-\sqrt{\mathsf{S}}x_\ell|^2 - \ln \mathbb{E}_\Theta[e^{2\Re\{(y-\sqrt{\mathsf{S}}x_\ell)\sqrt{\mathsf{I}}e^{-j\Theta}}]\Big)$$
$$= \arg\min_{\ell\in[1:N]} \Big(|y-\sqrt{\mathsf{S}}x_\ell|^2 - \ln I_0(2\sqrt{\mathsf{I}}|y-\sqrt{\mathsf{S}}x_\ell|)\Big),$$

where I_0 is the modified Bessel function of the first kind of order zero, which satisfies [24, eq(9.7.1)]

$$I_0(z) = \frac{e^z}{\sqrt{2\pi z}}\Big(1+o(1)\Big) \text{ for } z\to\infty,\ z\in\mathbb{R}^+. \quad (2)$$

For simplicity in the following we consider the BPSK case, that is, $\mathcal{X}=\{+1,-1\}$ with equal probability, but the analysis extends to a general real-valued modulation scheme. In this case the MAP implements the function

$$\hat{B}(y) = \begin{cases} 0 & \mathsf{LLR}(y) > 0 \\ 1 & \mathsf{LLR}(y) \le 0 \end{cases} : \quad (3)$$

where the log-likelihood ratio $\mathsf{LLR}(y)$ is

$$\mathsf{LLR}(y) = \ln\frac{\Pr[B=0|Y=y]}{\Pr[B=1|Y=y]} = \ln\frac{f_{Y|B}(y|0)}{f_{Y|B}(y|1)} \quad (4)$$
$$= \ln\frac{\mathbb{E}\Big[f_{Y|B,\Theta}(y|0,\Theta)\Big]}{\mathbb{E}\Big[f_{Y|B,\Theta}(y|1,\Theta)\Big]} \quad (5)$$
$$= \Big(|y-\sqrt{\mathsf{S}}|^2 - \ln I_0(2\sqrt{\mathsf{I}}|y-\sqrt{\mathsf{S}}|)\Big) \quad (6)$$
$$- \Big(|y+\sqrt{\mathsf{S}}|^2 - \ln I_0(2\sqrt{\mathsf{I}}|y+\sqrt{\mathsf{S}}|)\Big) \quad (7)$$
$$= 4\sqrt{\mathsf{S}}y_{\rm re} + g(y), \quad (8)$$

with $y_{\rm re} := \Re\{y\},\ y_{\rm im} := \Im\{y\}$, and $g(y)$ is expressed as

$$g(y) = \ln I_0\Big(2\sqrt{\mathsf{I}}\ |y-\sqrt{\mathsf{S}}|\Big) - \ln I_0\Big(2\sqrt{\mathsf{I}}\ |y+\sqrt{\mathsf{S}}|\Big) \quad (9)$$
$$= 2\sqrt{\mathsf{I}}\ (|y-\sqrt{\mathsf{S}}| - |y+\sqrt{\mathsf{S}}|) + o(1) \quad (10)$$
$$= -4\sqrt{\mathsf{S}}y_{\rm re}\ \cdot \frac{2\sqrt{\mathsf{I}}}{\sqrt{D_1}+\sqrt{D_2}} + o(1), \quad (11)$$
$$D_1 := |y-\sqrt{\mathsf{S}}|^2 = (y_{\rm re}-\sqrt{\mathsf{S}})^2 + y_{\rm im}^2, \quad (12)$$
$$D_2 := |y+\sqrt{\mathsf{S}}|^2 = (y_{\rm re}+\sqrt{\mathsf{S}})^2 + y_{\rm im}^2, \quad (13)$$

where in (10) we used the asymptotic expansion in (2) where it is assumed that $\mathsf{I} \gg \mathsf{S} \gg 1$ and $\mathsf{I}\to\infty$, and thus as a result, the optimal LLR can be approximated as

$$\mathsf{LLR}(y) \approx \underline{\mathsf{LLR}}(y) := 4\sqrt{\mathsf{S}}\ y_{\rm re}\Big(1-\frac{2\sqrt{\mathsf{I}}}{\sqrt{D_1}+\sqrt{D_2}}\Big), \quad (14)$$

where D_1 and D_2 are defined in (12) and (13), respectively. For the BER analysis we shall use the detector based on $\underline{\mathsf{LLR}}(y)$ to upper bound the optimal probability of error. Clearly, from (14) we see that the optimal LLR for $\mathsf{I} \gg \mathsf{S} \gg 1$ performs a subtraction that can be thought of as interference cancellation. It is instructive at this point to plot the optimal decoding regions based on (9), and to numerically evaluate the BER performance.

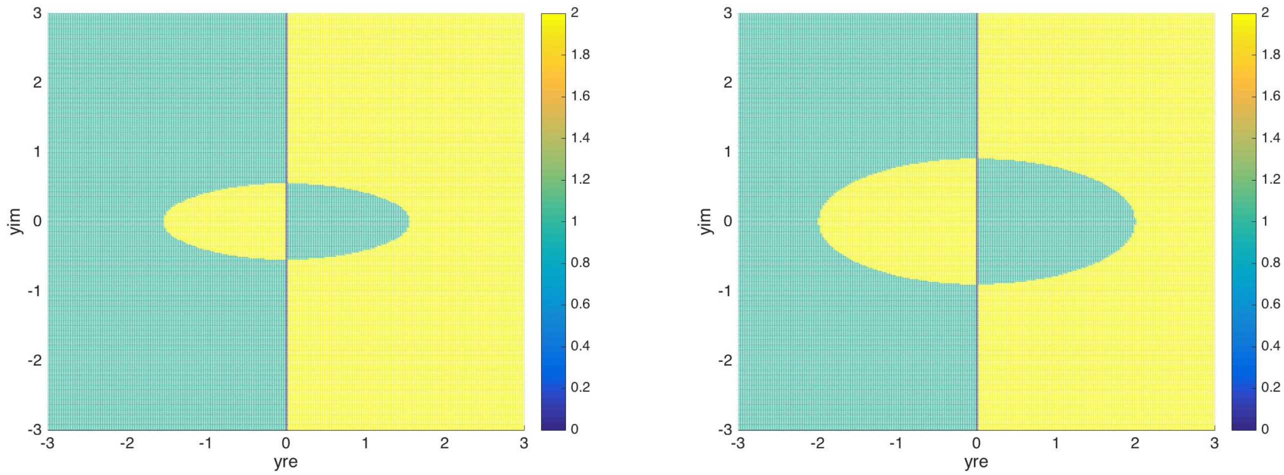
Fig. 1(a) shows the optimal decision regions based on (8), with $g(.)$ given in (9), in the plane $(y_{\rm re}, y_{\rm im})$ for $\mathsf{S}=5$ dB and $\mathsf{I}=6$ dB; while Fig. 1(b) shows the approximately optimal decision region based on (14). We would like to draw the reader's attention to the ellipsoidal-like region around the origin in Fig. 1. When SNR $\ge$ INR (a case not reported here for sake of space), such ellipsoidal-like region do not exist and the decision regions are determined solely by $\text{sign}(y_{\rm re})$, as for the case of AWGN only. Only when SNR $<$ INR those regions may appear; in this case it is interesting to note that within the ellipsoidal-like region the decision variable is $-\text{sign}(y_{\rm re})$, while outside the decision variable is $+\text{sign}(y_{\rm re})$. This may be seen analytically by working out the condition under which the expression in (14) has the same sign as $\text{sign}(y_{\rm re})$, which results in the ellipse $1 < \frac{y_{\rm re}^2}{\mathsf{I}} + \frac{y_{\rm im}^2}{\mathsf{I}-\mathsf{S}}$. The ellipsoidal-like region enlarges as INR increases; for large enough INR, the decision regions for the optimal and suboptimal decoders are almost undistinguishable, thus attesting to the tightness of the approximation in (14) for $\mathsf{I} \gg \mathsf{S} \gg 1$.

Fig. 2 shows the BER performance for the optimal decoder in (8), the suboptimal decoder in (14), and the baseline AWGN decoder based on $\text{sign}(y_{\rm re})$, as a function of I for a fixed S. Also shown are the asymptotic BER values for $\mathsf{I}=0$ and $\mathsf{I}\to\infty$. We point out that for higher S values the difference in performance between the optimal decoder in (9) and the suboptimal in (14) vanishes, as in this high SNR regime the approximation used for (11) becomes tighter and tighter. We notice that the optimal BER is not monotonic in I for a fixed S: it increases from its lower bound value $Q(\sqrt{2\mathsf{S}})$ when $\mathsf{I}=0$ up to about $\mathsf{I}=7.5$ dB (1.5 times the value of SNR in dB), then it decreases and flattens out to $\mathbb{E}[Q(\sqrt{2\mathsf{S}\cos^2(\Theta)})]$, as we shall show later. It can be seen that at around INR $\approx$ SNR, the probability of error reaches its maximum value. The interesting observation is hence that the BER for $\mathsf{I}\to\infty$ does not fall back to its $\mathsf{I}=0$ value, as one could have thought based on the intuition that a very strong interference could be estimated perfectly and thus subtracted off the received signal. If, at very high INR, the interference cancellation intuition is correct, then the 'error floor' in the BER means that the cancellation operation completely removes the interference but in the process also removes part of the useful signal. We mathematically formalize this intuition next.

The optimal BER can be bounded as follows

$$\Pr[\text{error}] = \Pr[\hat{X} \ne 1|X=1] \quad (15)$$
$$= \Pr[\mathsf{LLR}(Y) < 0|X=1] \quad (16)$$
$$\le \Pr[\underline{\mathsf{LLR}}(Y) < 0|X=1] \quad (17)$$
$$= \Pr\left[Y_{\rm re}\Big(1-\frac{2\sqrt{\mathsf{I}}}{\sqrt{D_1}+\sqrt{D_2}}\Big) < 0|X=1\right] \quad (18)$$
$$\approx \Pr[\cos^2(\Theta)\sqrt{\mathsf{S}} + Z' < 0] \quad (19)$$
$$= \mathbb{E}[Q(\sqrt{2\mathsf{S}\cos^2(\Theta)})], \quad (20)$$

where the approximation in (19) comes from considering the Taylor expansion of the optimal LLR at $\mathsf{I}\to\infty$ and finding that the equivalent real-valued noise Z' in this regime, for



(a) Optimal decision regions.

(b) Approximately optimal decision regions.

Fig. 1. Optimal and approximately optimal decision regions for BPSK signaling in the plane $(y_{\text{re}}, y_{\text{im}})$ for $\mathsf{S} = 5$ dB and $\mathsf{I} = 6$ dB.

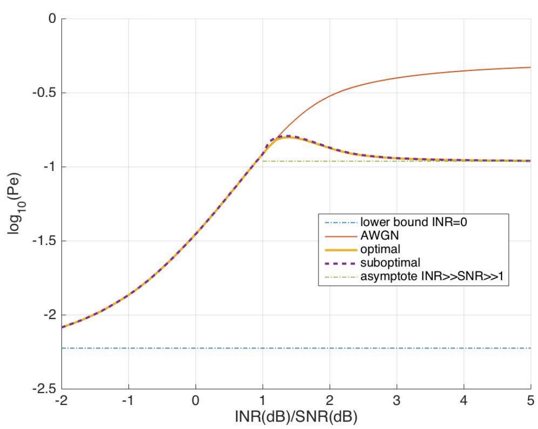


Fig. 2. BER for BPSK with $\mathsf{S} = 5$ dB vs. I in dB.

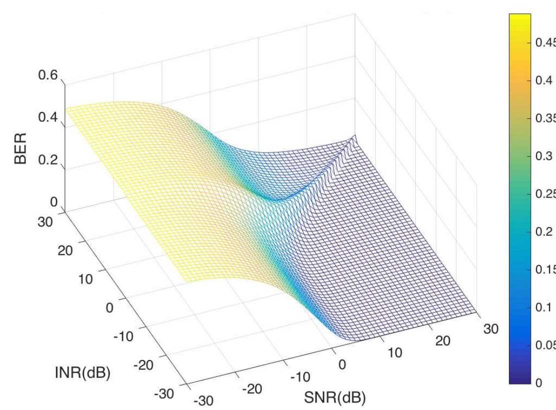


Fig. 3. BER vs. S and I in dB.

fixed Θ, is

$$Z' := \cos^2(\Theta) Z_{\text{re}} + \sin(\Theta)\cos(\Theta) Z_{\text{im}} \\ \sim \mathcal{N}(0, 1/2 \cdot \cos^2(\Theta))$$

and where $\mathcal{N}(\cdot,\cdot)$ denotes the real-valued Gaussian distribution and $Q(\cdot)$ the Q-function. The expression in (20) matches the numerical evaluation and shows that

$$\lim_{\mathsf{I}\to\infty} \Pr[\text{error}] = \mathbb{E}[Q(\sqrt{2\mathsf{S}\cos^2(\Theta)})]. \tag{21}$$

The BER expression in (21) can be interpreted as follows. The effect of a wide-band additive radar interference, of much larger power than the communication data signal, under the optimal MAP receiver is the same as that of a multiplicative/narrow-band fading known perfectly at the receiver. This provides a nice model for spectrum sharing that can benefit for the large body of work already done for narrow-band fading channels with fading known perfectly at the receiver [21]. The difference here, compared to standard fading models, is that the multiplicative interference is distributed as the cosine square of a uniformly distributed random variable —as opposed to Rayleigh or Rice distributed one.

Similarly to Fig. 2, which shows a log-scale BER vs. I in dB (normalized by S in dB), Fig. 3 shows the BER in linear scale versus S and I in dB for the optimal MAP decoder. It can be seen that at fixed SNR (dB), the BER increases with INR (dB) up to some point around INR (dB) $\approx$ SNR (dB) then the BER starts to decrease and flattens out to the asymptote given in (21). Note also the ridge around INR = SNR (an undesirable operating point) and that along this ridge, as expected, as SNR increases, the probability of error decreases.

As mentioned previously, the error rate analysis extends verbatim to a general real-valued constellation. Fig. 4 shows the symbol error rate vs. I of an 8-PAM modulation with $\mathsf{S} =$

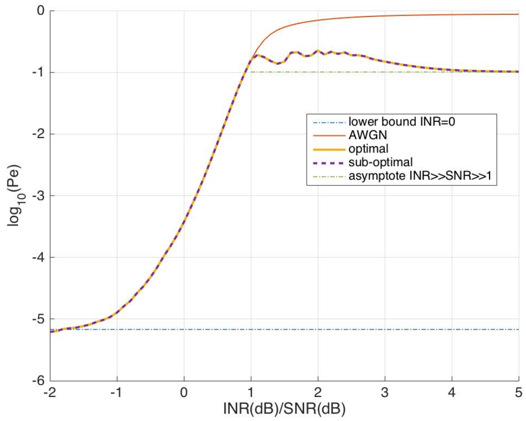


Fig. 4. BER for 8-PAM with $\mathsf{S} = 10$ dB vs. I in dB.

10 dB. It can be seen that the probability of error behaves as in AWGN for $\mathsf{I} \leq \mathsf{S}$ and that it has an 'error floor' for $\mathsf{I} \gg \mathsf{S} \gg 1$; this 'error floor' is exactly determined by the average error rate of the fading channel $\sqrt{2\cos^2(\Theta)}X + Z,\ Z \sim \mathcal{N}(0,1)$ and admits an expression similar to the one in (21) for the BPSK case. At $\mathsf{I}_{\mathrm{dB}} \approx 2\mathsf{S}_{\mathrm{dB}}$, the error probability attains its maximum value.

IV. Conclusions

In this paper, we studied the error rate performance of uncoded real-valued modulation schemes in a complex-valued Gaussian channel with additive constant-amplitude and random-phase radar-induced interference. We derived the expression for the optimal MAP decoder and we gave a tight, simple, closed-form asymptotic expression in the regime where the radar interference power is much larger than the SNR of the intended signal. Interestingly, we showed that the probability of error exhibits an irreducible error floor, which can be exactly characterized and behaves like a narrow-band fading channel with multiplicative fading that is perfectly known at the receiver.

Going forward, we believe that similar conclusions apply to general complex-valued (uncoded) modulation schemes. When considering actual wireless fading channels, this seems to imply that the channel model of interest, from the point of view of the communication receiver, is that of a narrowband fading model where the effective fading is the product of two terms: one corresponding to the classical (say Rayleigh or Rice) fading for the SNR, and the other due to the radar interference cancellation. Analysis of this model, as well as extensions to multi-channel/OFDM channel, channel-coded systems and multi-user channels, are interesting avenues of current investigation.

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