Handouts for NASIT 2016
The Interference Channel

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I. Detailed steps 1: Sato's outer bound for the interference channel

Theorem 1 (Sato's outer bound [1]). Let $\mathcal{R}(\tilde{p}(y_1, y_2|x_1, x_2))$ be the union over all $p(q)p(x_1|q)p(x_2|q)\tilde{p}(y_1, y_2|x_1, x_2)$ of

$$\begin{aligned} R_1 &\le I(X_1; Y_1|X_2, Q) \\ R_2 &\le I(X_2; Y_2|X_1, Q) \\ R_1 + R_2 &\le I(X_1, X_2; Y_1, Y_2|Q)\}. \end{aligned}$$

Then the intersection of the sets $\mathcal{R}(\tilde{p}(y_1, y_2|x_1, x_2))$ over all $\tilde{p}(y_1, y_2|x_1, x_2)$ with the same marginals as $p(y_1, y_2|x_1, x_2)$ is an outer bound for the DM-IC.

Proof. Intuitively, the first bound corresponds to a point-to-point bound between transmitter 1 and receiver 1 when receiver 1 has been given (as a "genie" or side information) the signal X_2 transmitter by transmitter 2. Similarly for the second bound. The third bound may be seen as a multiple-access channel bound, where the two receivers are permitted to cooperate in decoding both messages, i.e. they look and act like a single receiver with output (Y_1^n, Y_2^n) that must decode both messages. This is a straightforward proof, but we will go through all steps in detail just for practice.

$$\begin{aligned} nR_1 &= H(W_1) \\ &\overset{(a)}{=} H(W_1|W_2) \\ &\overset{(b)}{=} H(W_1|X_2^n) \\ &\overset{(c)}{=} H(W_1|Y_1^n, X_2^n) + I(W_1; Y_1^n|X_2^n) \\ &\overset{(d)}{\le} n\epsilon_1^n + I(W_1; Y_1^n|X_2^n) \\ &\overset{(e)}{=} n\epsilon_1^n + I(X_1^n; Y_1^n|X_2^n) \\ &\overset{(f)}{\le} n\epsilon_1^n + \sum_{j=1}^{n} I(X_{1j}; Y_{1j}|X_{2j}) \\ &\overset{(g)}{=} n\epsilon_1^n + nI(X_1; Y_1|X_2, Q), \end{aligned}$$

where: (a) follows by independence of W_1 and W_2, (b) follows by the fact that X_2^n is a function of W_2, (c) follows by definition of mutual information, (d) follows by Fano's inequality, i.e. in a converse, one can only use the problem statement and the fact that we are given a code whose probability of error $\to 0$ as $n \to \infty$. In this case we know that $P_{e1} := \Pr\{\widehat{W_1} \ne W_1\} \to 0$ and $P_{e2} := \Pr\{\widehat{W_2} \ne W_2\} \to 0$ as $n \to \infty$ since the average error $P_e := \Pr\{(\widehat{W_1}, \widehat{W_2}) \ne (W_1, W_2)\}$ goes to zero. (e) follows as

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X_1^n is a function of W_1, and (f) follows by

$$
\begin{aligned}
I(X_1^n;Y_1^n|X_2^n) &\stackrel{(h)}{=} \sum_{j=1}^{n} I(X_{1j};Y_1^n|X_2^n, X_{11}, \cdots X_{1(j-1)}) \\
&\stackrel{(i)}{\leq} \sum_{j=1}^{n} I(X_{1j};Y_{1j}|X_2^n, X_{11}, \cdots X_{1(j-1)}) \\
&\stackrel{(j)}{=} \sum_{j=1}^{n} H(Y_{1j}|X_2^n, X_{11}, \cdots X_{1(j-1)}) - H(Y_{1j}|X_2^n, X_{11}, \cdots X_{1(j-1)}, X_{1j}) \\
&\stackrel{(k)}{\leq} \sum_{j=1}^{n} H(Y_{1j}|X_{2j}) - H(Y_{1j}|X_2^n, X_{11}, \cdots X_{1(j-1)}, X_{1j}) \\
&\stackrel{(l)}{=} \sum_{j=1}^{n} H(Y_{1j}|X_{2j}) - H(Y_{1j}|X_{2j}, X_{1j})
\end{aligned}
$$

where (h) follows by the chain rule for mutual information, (i) since we have dropped terms in the mutual information (non-negativity of mutual information), (j) by definition, (k) as conditioning reduces entropy, (l) (KEY STEP) by the memorylessness of the channel – this is where we needed equality, in the negative subtraction terms when we try to single-letterize. Finally, (g) follows by taking Q to be a time-sharing random variable independent of all other random variables in the channel (hence it is also an auxiliary random variable as it does not depend on channel random variables), uniformly distributed on $[1:n]$. By extension, for arbitrary $p(q)$, where Q takes on values in some finite set we have the Theorem.

II. Detailed steps 2: Strong interference converse

Theorem (capacity region in strong interference [2]). The capacity region of the interference channel $(\mathcal{X}_1 \times \mathcal{X}_2, p(y_1,y_2|x_1,x_2), \mathcal{Y}_1 \times \mathcal{Y}_2)$in strong interference is the set of rate pairs (R_1, R_2) such that

$$R_1 \leq I(X_1;Y_1|X_2,Q) \tag{1}$$
$$R_2 \leq I(X_2;Y_2|X_1,Q) \tag{2}$$
$$R_1 + R_2 \leq \min\{I(X_1,X_2;Y_1|Q), I(X_1,X_2;Y_2|Q)\} \tag{3}$$

for some $p(q,x_1,x_2) = p(q)p(x_1|q)p(x_2|q)$ where $|\mathcal{Q}| \leq 4$.

Proof of converse. First two single rate bound inequalities follow by the "basic genie" outer bound. For the last inequality, we only need to show one of the inequalities by symmetry. The trick is to get a sum-rate bound in terms of a mutual information term with only one output. This is done by using the strong interference condition.

$$
\begin{aligned}
n(R_1+R_2) &= H(W_1) + H(W_2) \\
&\stackrel{(a)}{\leq} I(W_1;Y_1^n) + I(W_2;Y_2^n) + n\epsilon_n \\
&\stackrel{(b)}{\leq} I(X_1^n;Y_1^n) + I(X_2^n;Y_2^n) + n\epsilon_n \\
&\stackrel{(c)}{\leq} I(X_1^n;Y_1^n|X_2^n) + I(X_2^n;Y_2^n) + n\epsilon_n \\
&\stackrel{(d)}{\leq} I(X_1^n;Y_2^n|X_2^n) + I(X_2^n;Y_2^n) + n\epsilon_n \\
&\stackrel{(e)}{=} I(X_1^n,X_2^n;Y_2^n) + n\epsilon_n \\
&\stackrel{(f)}{\leq} \sum_{i]1}^{n} I(X_{1i},X_{2i};Y_{2i}) + n\epsilon_n \\
&\stackrel{(g)}{=} nI(X_1,X_2;Y_2|Q) + n\epsilon_n
\end{aligned}
$$

where (a) follows by Fano's inequality, (b) by the Markov chain $W_i \to X_i^n \to Y_i^n$, (c) $I(X_1^n;Y_1^n) \leq I(X_1^n;Y_1^n,X_2^n) = I(X_1^n;X_2^n)+I(X_1;Y_1^n|X_2^n) = I(X_1^n;Y_1^n|X_2^n)$, (d) by a Lemma from [2] that states that for a DM-IC under strong interference, $I(X_1^n;Y_1^n|X_2^n) \leq I(X_1^n;Y_2^n|X_2^n)$ for all $p(x_1^n)p(x_2^n)$ and all $n \geq 1$, (e) by definition, (f) in a similar fashion as in the previous single-letterization, an (g) by introducing the time-sharing auxiliary random variable Q.

	m_{10}	m_{20}	m_{11}	Joint pmf
1	1	1	1	$p(u_1^n, x_1^n)p(u_2^n)p(y_1^n\|x_1^n, u_2^n)$
2	1	1	$*$	$p(u_1^n, x_1^n)p(u_2^n)p(y_1^n\|u_1^n, u_2^n)$
3	$*$	1	$*$	$p(u_1^n, x_1^n)p(u_2^n)p(y_1^n\|u_2^n)$
4	$*$	1	1	$p(u_1^n, x_1^n)p(u_2^n)p(y_1^n\|u_2^n)$
5	1	$*$	$*$	$p(u_1^n, x_1^n)p(u_2^n)p(y_1^n\|u_1^n)$
6	$*$	$*$	1	$p(u_1^n, x_1^n)p(u_2^n)p(y_1^n)$
7	$*$	$*$	$*$	$p(u_1^n, x_1^n)p(u_2^n)p(y_1^n)$
8	1	$*$	1	$p(u_1^n, x_1^n)p(u_2^n)p(y_1^n\|x_1^n)$

Fig. 1. Error events, taken directly from [3].

III. Detailed steps 3: Probability of error in Han+Kobayashi achievable rate region

This error analysis is the same as that given in Abbas El Gamal and Young-Han Kim's excellent book "Network Information Theory" [3]. I am just extracting and expanding upon some of the parts here. In short, they unify and shorten many network information theory achievability proofs through the use of the "packing lemma", which reads as follows:

Packing Lemma [3]. Let $(U,X,Y) \sim p(u,x,y)$. Let $(\tilde{U^n}, \tilde{Y^n}) \sim p(\tilde{u^n}, \tilde{y^n})$ be a pair of arbitrarily distributed random sequences (not necessarily according to $\prod_{I=1}^n p_{U,Y}(\tilde{u}_i, \tilde{y}_i)$). Let $X^n(m), m \in \mathcal{A}$, where $|\mathcal{A}| \leq 2^{nR}$, be random sequences, each distributed according to $\prod_{i=1}^n p_{X|U}(x_i|\tilde{u}_1)$. Assume that $X^n(m), m \in \mathcal{A}$, is pairwise conditionally independent of $\tilde{Y^n}$ given $\tilde{U^n}$, but is arbitrarily dependent on other $X^n(m)$ sequences. Then, there exists $\delta(\epsilon) \to 0$ as $\epsilon \to 0$ such that

$$\Pr\{(\tilde{U^n}, X^n(m), \tilde{Y^n}) \in \mathcal{T}_\epsilon^n\} \to 0 \text{ as } n \to \infty \text{ if } R < I(X;Y|U) - \delta(\epsilon),$$

where $\mathcal{T}_\epsilon^{(n)}$ is defined as the typical set

$$\mathcal{T}_\epsilon^{(n)} = \mathcal{T}_\epsilon^{(n)}(U,X,Y) := \{(u^n, x^n, y^n) : |\pi(u,x,y|u^n,x^n,y^n) - p(u,x,y)| \leq \epsilon \cdot p(u,x,y)\},$$

where

$$\pi(u,x,y|u^n,x^n,y^n) = \frac{|\{i : (u_i,x_i,y_i) = (u,x,y)|}{n} \text{ for } (u,x,y) \in \mathcal{U} \times \mathcal{X} \times \mathcal{Y}$$

Probability of error of the Han+Kobayashi achievability scheme. Recall that message $w_{1c}, w_{1p}, w_{2c}, w_{2p}$ have rates $R_{1c}, R_{1p}, R_{2c}, R_{2p}$, respectively. Assume (WLOG) that message pairs $(w_{1c}, w_{1p}) = (1,1)$ and $(w_{2c}, w_{2p}) = (1,1)$ are sent. We look at each decoder separately and bound the average (over all codes randomly generated as such, and passed through a memoryless channel ???) probability of error. We will first show that the following rates are achievable, and then use Fourier-Motzkin elimination to show the final form (in terms of R_1 and R_2 only).

$$\begin{aligned}
R_{1p} &\leq I(X_1;Y_1|U_1,U_2,Q)\\
R_{1p} + R_{1c} &\leq I(X_1;Y_1|U_2,Q)\\
R_{1p} + R_{2c} &\leq I(X_1,U_2;Y_1|U_1,Q)\\
R_{1p} + R_{1c} + R_{2c} &\leq I(X_1,U_2;Y_1|Q)\\
R_{2p} &\leq I(X_1;Y_2|U_1,U_2,Q)\\
R_{2p} + R_{2c} &\leq I(X_2;Y_2|U_1,Q)\\
R_{2p} + R_{1c} &\leq I(X_2,U_1;Y_2|U_2,Q)\\
R_{2p} + R_{2c} + R_{1c} &\leq I(X_2,U_1;Y_2|Q)
\end{aligned}$$

We look at the different types of errors that can occur. El Gamal and Kim very nicely enumerate all the possible errors in a table, along with the output distribution that is induced with this type of error (replace $w_{1c} = m_{10}$, $w_{1p} = m_{11}$, $w_{2c} = m_{20}$ and $w_{2p} = m_{22}$ to go from my notation to theirs). What do you notice? Case 8 is not an error, and cases 3,4 and 6,7, have the same pmf, and case 1 results only in an error if the true messages are not jointly typical with the output. Hence, we are left with the following 5 errors:

The remainder of the proof follows by simple arguments; we cut-and-paste from [3] (the slides), which succinctly enumerates the possible errors.

Each error term may then be bounded as follows (remember all we need to do is show that the probability of error vanishes as $n \to \infty$, which again, taken directly from [3] yields:

- We are left with only 5 error events:

$$
\begin{aligned}
\mathcal{E}_{10} &:= \{(Q^n, U_1^n(1), U_2^n(1), X_1^n(1,1), Y_1^n) \notin \mathcal{T}_\epsilon^{(n)}\}, \\
\mathcal{E}_{11} &:= \{(Q^n, U_1^n(1), U_2^n(1), X_1^n(1, m_{11}), Y_1^n) \in \mathcal{T}_\epsilon^{(n)} \\
&\qquad\qquad \text{for some } m_{11} \neq 1\}, \\
\mathcal{E}_{12} &:= \{(Q^n, U_1^n(m_{10}), U_2^n(1), X_1^n(m_{10}, m_{11}), Y_1^n) \in \mathcal{T}_\epsilon^{(n)} \\
&\qquad\qquad \text{for some } m_{10} \neq 1, m_{11}\}, \\
\mathcal{E}_{13} &:= \{(Q^n, U_1^n(1), U_2^n(m_{20}), X_1^n(1, m_{11}), Y_1^n) \in \mathcal{T}_\epsilon^{(n)} \\
&\qquad\qquad \text{for some } m_{20} \neq 1, m_{11} \neq 1\}, \\
\mathcal{E}_{14} &:= \{(Q^n, U_1^n(m_{10}), U_2^n(m_{20}), X_1^n(m_{10}, m_{11}), Y_1^n) \in \mathcal{T}_\epsilon^{(n)} \\
&\qquad\qquad \text{for some } m_{10} \neq 1, m_{20} \neq 1, m_{11}\}
\end{aligned}
$$

Then, the average probability of error for decoder 1 is

$$\mathsf{P}(\mathcal{E}_1) \leq \sum_{j=0}^{4} \mathsf{P}(\mathcal{E}_{1j})$$

Fig. 2. Error events, taken directly from [3]

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- Now, we bound each probability of error term
  1. By the LLN, $\mathsf{P}(\mathcal{E}_{10}) \to 0$ as $n \to \infty$
  2. By the packing lemma, $\mathsf{P}(\mathcal{E}_{11}) \to 0$ as $n \to \infty$ if $R_{11} < I(X_1; Y_1 | U_1, U_2, Q) - \delta(\epsilon)$
  3. By the packing lemma, $\mathsf{P}(\mathcal{E}_{12}) \to 0$ as $n \to \infty$ if $R_{11} + R_{10} < I(X_1; Y_1 | U_2, Q) - \delta(\epsilon)$
  4. By the packing lemma, $\mathsf{P}(\mathcal{E}_{13}) \to 0$ as $n \to \infty$ if $R_{11} + R_{20} < I(X_1, U_2; Y_1 | U_1, Q) - \delta(\epsilon)$
  5. By the packing lemma, $\mathsf{P}(\mathcal{E}_{14}) \to 0$ as $n \to \infty$ if $R_{11} + R_{10} + R_{20} < I(X_1, U_2; Y_1 | Q) - \delta(\epsilon)$
- The average probability of error for decoder 2 can be bounded similarly
- Finally, we use the Fourier–Motzkin procedure with the constraints $R_{j0} = R_j - R_{jj}, 0 \leq R_{jj} \leq R_j$ for $j = 1, 2$, to eliminate R_{11}, R_{22} and obtain the region given in the theorem (see Appendix D for details)

Fig. 3. Error events, taken directly from [3]

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IV. Detailed steps 4: Converse for capacity region of class of deterministic IC and semi-deterministic IC

Theorem (outer bound of semi-deterministic IC) Every achievable rate pair (R_1, R_2) must satisfy

$$R_1 \le H(Y_1|X_2,Q) - H(T_2|X_2) \tag{4}$$
$$R_2 \le H(Y_2|X_1,Q) - H(T_1|X_1) \tag{5}$$
$$R_1 + R_2 \le H(Y_1|Q) + H(Y_2|U_2,X_1,Q) - H(T_1|X_1) - H(T_2|X_2) \tag{6}$$
$$R_1 + R_2 \le H(Y_1|U_1,X_2,Q) + H(Y_2|Q) - H(T_1|X_1) - H(T_2|X_2) \tag{7}$$
$$R_1 + R_2 \le H(Y_1|U_1,Q) + H(Y_2|U_2,Q) - H(T_1|X_1) - H(T_2|X_2) \tag{8}$$
$$2R_1 + R_2 \le H(Y_1|Q) + H(Y_1|U_1,X_2,Q) + H(Y_2|U_2,Q) - H(T_1|X_1) - 2H(T_2|X_2) \tag{9}$$
$$R_1 + 2R_2 \le H(Y_2|Q) + H(Y_2|U_2,X_1,Q) + H(Y_1|U_1,Q) - 2H(T_1|X_1) - H(T_2|X_2) \tag{10}$$

for some $p(q,x_1,x_2) = p(q)p(x_1|q)p(x_2|q)$ and $p(u_1,u_2|q,x_1,x_2) = p_{T_1|X_1}(u_1|x_1)p_{T_2|X_2}(u_2|x_2)$.

This converse is interesting as it showcases one of the major difficulties in converses – single-letterizing them. A single-letterization is nice because not only does it "look" nicer and more intuitive, but the optimization to be carried out is often easier. Nowadays, it is sort of assumed that unless ypu have a single-letter expression for capacity, that capacity is unknown. This may change in the future as computational power increases, or if we find eaier ways to optimize multi-letter expressions, but this is mere speculation on my part.

Let us attack the converse, which is taken from the simple and elegant [3], but originally appeared in [4], which is essentially an extension of the ideas in [5]. Consider a sequence of $(2^{nR_1}, 2^{nR_2})$ codes with $P_e^{(n)} \to 0$. Futhermore, let $X_1^n, X_2^n, T_1^n, T_2^n, Y_1^n, Y_2^n$ denote the random variables results from encoding and transmitting the independent messages W_1 and W_2. Define random variables U_1^n and U_2^n such that U_{ji} is jointly distributed with X_{ji} according to $p_{T_j|X_j}(u|x_{ji})$, conditionally independent of T_{ji} given X_{ji} for every $j = 1, 2$ and every $i \in [1:n]$.

Fano's inequality, for $j = 1, 2$, yields:

$$\begin{aligned} nR_j = H(W_j) &= I(W_j; Y_j^n) + H(W_j|Y_j^n) \\ &\le I(W_j; Y_j^n) + n\epsilon_n \\ &\le I(X_j^n; Y_j^n) + n\epsilon_n \end{aligned}$$

This is a multi-letter outer bound for the capacity region. We now look for non-trivial single-letter outer bounds. The innovative trick is to find several partially single-letterized outer bounds on R_1 and R_2 separately, some of which contain a multi-letter term, and then to take linear combinations of these outer bounds to obtain single-letter bounds. We omit the $+n\epsilon_n$ terms in the following for simplicity (since this tends to 0 anyhow as $n \to \infty$).

Bound A1:

$$\begin{aligned} nR_1 &\le I(X_1^n; Y_1^n) \\ &= H(Y_1^n) - H(Y_1^n|X_1^n) \\ &= H(Y_1^n) - H(T_2^n|X_1^n) \\ &= H(Y_1^n) - H(T_2^n) \\ &\le \sum_{i=1}^n H(Y_{1i}) - \boxed{H(T_2^n)} \end{aligned}$$

Bound B1: (genie at Rx 1 of U_1^n, X_2^n)

$$\begin{aligned} nR_1 &\le I(X_1^n; Y_1^n, U_1^n, X_2^n) \\ &= I(X_1^n; U_1^n) + I(X_1^n; X_2^n|U_1^n) + I(X_1^n; Y_1^n|U_1^n, X_2^n) \\ &= H(U_1^n) - H(U_1^n|X_1^n) + H(Y_1^n|U_1^n, X_2^n) - H(Y_1^n|X_1^n, U_1^n, X_2^n) \\ &\overset{(a)}{=} H(T_1^n) - H(U_1^n|X_1^n) + H(Y_1^n|U_1^n, X_2^n) - H(T_2^n|X_2^n) \\ &\le \boxed{H(T_1^n)} - \sum_{i=1}^n H(U_{1i}|X_{1i}) + \sum_{i=1}^n H(Y_{1i}|U_{1i}, X_{2i}) - \sum_{i=1}^n H(T_{2i}|X_{2i}) \end{aligned}$$

Bound C1: (genie at Rx 1 of U_1^n)

$$\begin{aligned}
nR_1 &\leq I(X_1^n; Y_1^n, U_1^n) \\
&= I(X_1^n; U_1^n) + I(X_1^n; Y_1^n | U_1^n) \\
&= H(U_1^n) - H(U_1^n | X_1^n) + H(Y_1^n | U_1^n) - H(Y_1^n | X_1^n, U_1^n) \\
&= H(T_1^n) - H(U_1^n | X_1^n) + H(Y_1^n | U_1^n) - H(T_2^n) \\
&\leq \boxed{H(T_1^n)} - \boxed{H(T_2^n)} - \sum_{i=1}^{n} H(U_{1i} | X_{1i}) + \sum_{i=1}^{n} H(Y_{1i} | U_{1i})
\end{aligned}$$

Bound D1: (genie at Rx 1 of X_2^n)

$$\begin{aligned}
nR_1 &\leq I(X_1^n; Y_1^n, X_2^n) \\
&= I(X_1^n; X_2^n) + I(X_1^n; Y_1^n | X_2^n) \\
&= H(Y_1^n | X_2^n) - H(Y_1^n | X_1^n, X_2^n) \\
&= H(Y_1^n | X_2^n) - H(T_2^n | X_2^n) \\
&\leq \sum_{i=1}^{n} H(Y_{1i} | X_{2i}) - \sum_{i=1}^{n} H(T_{2i} | X_{2i})
\end{aligned}$$

By symmetry, we have the following 4 bounds on R_2:

Bound A2:

$$nR_2 \leq \sum_{i=1}^{n} H(Y_{2i}) - \boxed{H(T_1^n)}$$

Bound B2: (genie at Rx 2 of U_2^n, X_1^n)

$$nR_2 \leq \boxed{H(T_2^n)} - \sum_{i=1}^{n} H(U_{2i} | X_{2i}) + \sum_{i=1}^{n} H(Y_{2i} | U_{2i}, X_{1i}) - \sum_{i=1}^{n} H(T_{1i} | X_{1i})$$

Bound C2: (genie at Rx 2 of U_2^n)

$$nR_2 \leq \boxed{H(T_2^n)} - \boxed{H(T_1^n)} - \sum_{i=1}^{n} H(U_{2i} | X_{2i}) + \sum_{i=1}^{n} H(Y_{2i} | U_{2i})$$

Bound D2: (genie at Rx 2 of X_1^n)

$$nR_2 \leq \sum_{i=1}^{n} H(Y_{2i} | X_{1i}) - \sum_{i=1}^{n} H(T_{1i} | X_{1i})$$

Then we obtain the bounds in the semi-deterministic channel outer bound theorem by looking at the following linear combinations, and using a time-sharing random variable uniformly distributed on $[1:n]$. Specifically, (4) is obtained from D1, (5) from D2, (6) from A1+B2, (7) from B1+A2, (8) from C1+C2, (9) from A1+B1+C2, and (10) from A2+B2+C1, by additionally noting that $H(U_2|X_2) = H(T_1|X_2)$ and $H(U_1|X_1) = H(T_1|X_2)$ based on the construction of U_1 and U_2.

Summary of bounds:

$$\textbf{Bound A1:}\quad nR_1 \leq \sum_{i=1}^{n} H(Y_{1i}) - \boxed{H(T_2^n)}$$

$$\textbf{Bound B1:}\quad nR_1 \leq \boxed{H(T_1^n)} - \sum_{i=1}^{n} H(U_{1i}|X_{1i}) + \sum_{i=1}^{n} H(Y_{1i}|U_{1i}, X_{2i}) - \sum_{i=1}^{n} H(T_{2i}|X_{2i})$$

$$\textbf{Bound C1:}\quad nR_1 \leq \boxed{H(T_1^n)} - \boxed{H(T_2^n)} - \sum_{i=1}^{n} H(U_{1i}|X_{1i}) + \sum_{i=1}^{n} H(Y_{1i}|U_{1i})$$

$$\textbf{Bound D1:}\quad nR_1 \leq \sum_{i=1}^{n} H(Y_{1i}|X_{2i}) - \sum_{i=1}^{n} H(T_{2i}|X_{2i})$$

$$\textbf{Bound A2:}\quad nR_2 \leq \sum_{i=1}^{n} H(Y_{2i}) - \boxed{H(T_1^n)}$$

$$\textbf{Bound B2:}\quad nR_2 \leq \boxed{H(T_2^n)} - \sum_{i=1}^{n} H(U_{2i}|X_{2i}) + \sum_{i=1}^{n} H(Y_{2i}|U_{2i}, X_{1i}) - \sum_{i=1}^{n} H(T_{1i}|X_{1i})$$

$$\textbf{Bound C2:}\quad nR_2 \leq \boxed{H(T_2^n)} - \boxed{H(T_1^n)} - \sum_{i=1}^{n} H(U_{2i}|X_{2i}) + \sum_{i=1}^{n} H(Y_{2i}|U_{2i})$$

$$\textbf{Bound D2:}\quad nR_2 \leq \sum_{i=1}^{n} H(Y_{2i}|X_{1i}) - \sum_{i=1}^{n} H(T_{1i}|X_{1i})$$

REFERENCES

[1] S. H, "Two-user communication channels," IEEE Trans. Inf. Theory, vol. 23, 1977.

[2] M.H.M. Costa and A. El Gamal, "The capacity region of the discrete memoryless interference channel with strong interference," IEEE Trans. Inf. Theory, vol. 33, pp. 710–711, Sep. 1987.

[3] A. El Gamal and Y.-H. Kim, Network Information Theory. Cambridge University Press, 2011.

[4] E. Telatar and D. Tse, "Bounds on the capacity region of a class of interference channels," in Proc. IEEE Int. Symp. Inf. Theory. IEEE, 2008, pp. 2871–2874.

[5] A. El Gamal and M.H.M. Costa, "The capacity region of a class of deterministic interference channels," IEEE Trans. Inf. Theory, vol. 28, no. 2, pp. 343–346, Mar. 1982.