On constant gaps for the K-pair user two-way Gaussian interference channel with interaction

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***Abstract*—In a K-pair-user two-way Gaussian interference channel (IC), $2K$ messages and $2K$ transmitters/receivers form a Gaussian K-user IC in the forward direction (K messages) and another Gaussian K-user IC in the backward direction (K messages) which operate simultaneously in full-duplex mode. All nodes are permitted to interact, i.e. adapt current channel inputs to past received signals. We derive a new sum-rate outer bound for linear deterministic and Gaussian noise channels allowing for interaction, but show that for symmetric scenarios and certain interference regimes (moderately weak and strong), non-interactive schemes achieve to within a constant gap of this outer bound. That is, interaction for symmetric channels in certain interference regimes may only improve the sum-rate by a constant number of bits per channel use.**

I. Introduction

Shannon first proposed and studied the point-to-point two-way channel in 1961 [1], but relatively few results on two-way point-to-point channels, and even fewer on two-way networks, have emerged in recent years. This may be in part due to the difficulty which arises when one permits *interaction* between nodes, i.e., each node can adapt its channel inputs to their past received signals. How to best adapt, in order to characterize the capacity of two-way communications, is a very challenging question, and the capacity of two-way channels in general remains open.

However, capacity is known for several point-to-point two-way channel models such as the two-way modulo 2 binary adder channel and two-way Gaussian channel [2], as well as recently studied multi-user two-way channels, for several specific cases [3]. It appears that capacity is known only for specific two-way channels in which interaction between nodes is shown to be useless, i.e., it cannot increase the capacity, and thus the capacities of these (multi-user) two-way channels are equivalent to the capacities of two simultaneously operating one-way channels.

One multi-user two-way channel first studied by the authors is the two-way interference channel [3]–[6], in which there are 4 independent messages: two-messages to be transmitted over an interference channel (IC) in the $\rightarrow$ direction simultaneously with two-messages to be transmitted over an in-band IC in the $\leftarrow$ direction. All 4 nodes in this network act as both sources and destinations of messages, which allows for interaction between the nodes. The capacity of the two-way interference channel, like the one-way interference channel, is still open, but is known for certain classes of deterministic channels [3], [4] and to within a constant gap for the Gaussian channel in certain parameter regimes and under certain adaptation constraints [3], [5].

In this work, we first propose and study a natural extension of the (2-pair-user) two-way IC: the K-pair-user two-way IC where there are $2K$ messages and $2K$ users forming a K-user IC (K messages) in the forward direction and another K-user IC in the backward direction (K messages). Again, all nodes may employ interaction – i.e. signals may be a function of previously received outputs. Compared to the 2-pair-user IC, each user in the K-pair-user two-way IC suffers interference from the $K-1$ users on the opposite side, as well as possibly from the $K-1$ users on the same side due to the interaction between users. Thus, received signals may in general be combinations of all $2K$ messages. We note that the self-interference signals can be easily subtracted off at receivers in this theoretical work since each user knows its own signal and the considered Gaussian channel is additive.

We derive new outer bounds for the symmetric K-pair-user two-way linear deterministic and Gaussian ICs. For the linear deterministic channel model [7], we show the sum-capacity in the moderately weak and strong interference regimes, and this corresponds to the sum-capacity when no adaptation is permitted (i.e. the "W" curve for two one-way K-user ICs). Achievability follows from the existing non-interactive scheme for the one-way K-user linear deterministic IC [8]. For the Gaussian model, again for the moderately weak and strong interference regimes, we show that the symmetric sum-capacity is to within a constant gap of two non-interactive outer bounds of two simultaneous one-way K-user Gaussian ICs, which in turn is to within a constant gap of non-interactive achievability scheme for most channel gains (i.e. outside an outage set), as shown in [9]. The technique used for deriving outer bounds bears semblance to the proof of the outer bound for the one-way K-user Gaussian IC with feedback [10]. However, due to the additional messages, additional noise and adaptation in our channel model, the construction of some terms in the proof is non-trivial and novel.

The *one-way* K-user interference channel has been previously studied: in [11], the degrees of freedom were shown to be $K/2$; the generalized degrees of freedom was obtained in [8]; the approximate sum capacity of the symmetric K-User Gaussian IC was shown in [9]; and the feedback capacity was studied by [10]. We note that our work differs from prior work in that we consider an *interactive*, *full-duplex* two-way K-pair-user IC for the first time, and our results show that interaction

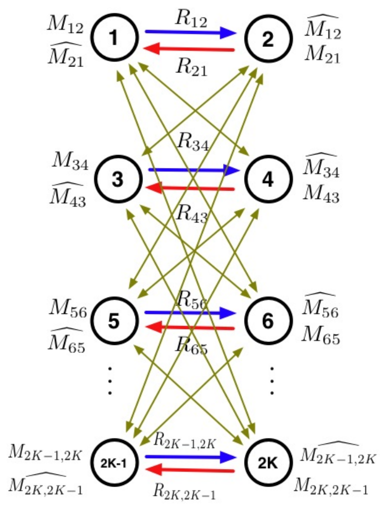

Fig. 1. K-pair-user two-way interference channel. M_{jk} denotes the message known at node j and desired at node k; $\widehat{M_{jk}}$ denotes that k would like to decode the message M_{jk} from node j.

again is useless or cannot significantly increase the *symmetric sum-capacity* in certain regimes.

II. System Model

In this section, we describe the K-pair-user two-way interference channel, and in particular, the Gaussian and linear deterministic channel models.

A. K-pair-user two-way interference channel

We consider a K-pair-user two-way interference channel as shown in Fig. 1, where there are $2K$ messages and $2K$ terminals forming a K-user IC in the $\rightarrow$ direction (K messages) and another K-user IC in the $\leftarrow$ direction (K messages). All nodes are able to operate in full-duplex mode, i.e. they can transmit and receive signals simultaneously.

The channel inputs and outputs of user $j \in \{1, 2, ..., 2K\}$ at discrete time i are $X_{j,i}$ and $Y_{j,i}$ that lie in alphabets $\mathcal{X}_j$ and $\mathcal{Y}_j$ respectively. The messages M_{jk} of rate R_{jk} from transmitter j to receiver k are uniformly distributed in $\{1, 2, \cdots 2^{nR_{jk}}\}$ for $j, k \in \{1, 2, ..., 2K\}$ and blocklength n. Let $A_j^i = (A_{j,1}, A_{j,2}, ..., A_{j,i})$, for any given time i. A node is said to employ *interaction* if the channel input at time i is a function of the previously received outputs, $X_{j,i} = f_j(M_{jk}, Y_j^{i-1})$, where f_j ($j \in \{1, 2, ..., 2K\}$) are deterministic functions. Receiver k uses a decoding function $d_k : \mathcal{Y}_k^n \rightarrow \widehat{\mathcal{M}}_{jk}$ to obtain an estimate $\widehat{M}_{jk}$ of the transmitted message M_{jk}. The capacity region is the closure of all rate tuples which simultaneously drive the probability that any of the estimated messages is not equal to the true message, to zero as $n \rightarrow \infty$.

B. Gaussian model

The K-pair-user Gaussian two-way interference channel at each channel use, is described by (with subscripts "o" for odd and "e" for even)

$$Y_o = \sum_{m=1}^{K} g_{2m,o} X_{2m} + Z_o, \quad o = 1, 3, ..., 2K-1 \tag{1}$$

$$Y_e = \sum_{m=1}^{K} g_{2m-1,e} X_{2m-1} + Z_e, \quad e = 2, 4, ..., 2K. \tag{2}$$

where $g_{jk}, j, k \in \{1, 2, ..., 2K\}$ is the channel coefficient from node j to node k, and the network is subject to complex Gaussian noise $Z_l \sim \mathcal{CN}(0, 1), l \in \{1, 2, ..., 2K\}$. Let P be the transmit power constraint at each user: $E[|X_l|^2] \leq P, l \in \{1, 2, ..., 2K\}$, and let $P = 1$ without loss of generality. Then define $\texttt{SNR}_{l,l+1} = |g_{l,l+1}|^2, \texttt{SNR}_{l+1,l} = |g_{l+1,1}|^2, l \in \{1, 2, ..., 2K\}$, and $\texttt{INR}_{jk} = |g_{jk}|^2$, for j, k in the appropriate sets that denote cross links between users. Note that we have removed the "self-interference" terms such as $g_{11}X_1$ in the expression of Y_1 (for example) since they can be easily subtracted off due to the additive nature of the channel.

Symmetric capacity. We are interested in the symmetric capacity when all the SNRs equal a given $\texttt{SNR}$, and all the INRs equal a given $\texttt{INR}$. We consider the per-user rates $R_{sym} = \frac{R_{12}+R_{34}}{2} = \frac{R_{21}+R_{43}}{2}$.

C. Linear deterministic model

For the linear deterministic model which models the Gaussian channel at high SNR, the channel inputs and outputs are binary vectors, and all addition is bit-wise and modulo 2. We define $n_{jk} = \lfloor \log g_{jk}^2 P \rfloor$ to indicate the number of signal bit levels from transmitter j to receiver k. Let S denote an $N \times N$ lower shift matrix, where $N = \max(n_{jk})$. Now the channel inputs/outputs relationship can be described as

$$Y_o = \sum_{m=1}^{K} S^{N-n_{2m,o}} X_{2m}, \quad o = 1, 3, ..., 2K-1 \tag{3}$$

$$Y_e = \sum_{m=1}^{K} S^{N-n_{2m-1,e}} X_{2m-1}, \quad e = 2, 4, ..., 2K. \tag{4}$$

III. Outer bounds and capacity/gap results

For the symmetric K-pair-user two-way interference channel with interaction in two "medium" interference regimes (to be specified in the theorem statements), we derive new outer bounds and demonstrate a capacity result for the linear deterministic and a constant gap result for the Gaussian model.

A. Capacity result for the linear deterministic model

We consider the symmetric case in which all direct links have the same number of signal bit levels p, and all cross links have the same number of signal bit levels q, and define $\alpha = q/p$.

Theorem 1: The symmetric sum-capacity of linear deterministic K-pair-user two-way interference channel with interaction when $2/3 < \alpha < 2, \alpha \neq 1$ is the rate which satisfies:

$$R_{sym} \leq \frac{1}{2}(\max(p, q) + [p - q]^+) \tag{5}$$

Proof: Achievability follows from the known non-adaptive scheme as in [8] (used in each direction separately). For the converse, valid for interactive, two-way channel models, let M_A denote all the messages except M_{12}, M_{34}. Then,

$$
\begin{aligned}
& n(R_{12}+R_{34}) \\
& \leq H(M_{12})+H(M_{34}) \\
& = H(M_{12}, M_{34}|M_A) \\
& \leq H(M_{12}, M_{34}, Y_2^n, Y_4^n|M_A) \\
& = H(Y_4^n|M_A)+H(M_{34}|M_A, Y_4^n) \\
& + H(Y_2^n|M_{34}, M_A, Y_4^n)+H(M_{12}|M_{34}, Y_2^n, Y_4^n, M_A) \\
& \leq H(Y_4^n)+H(M_{34}|Y_4^n)+H(Y_2^n|M_{34}, M_A, Y_4^n)+H(M_{12}|Y_2^n) \\
& \leq n(\max(p,q)+2\epsilon)+H(Y_2^n|M_{34}, M_A, Y_4^n).
\end{aligned}
$$

We proceed to bound the remaining entropy term:

$$
\begin{aligned}
& H(Y_2^n|M_{34}, M_A, Y_4^n) \\
& \leq H(Y_2^n, Y_3^n, Y_5^n, ..., Y_{2K-1}^n|M_{34}, M_A, Y_4^n) \\
& = \sum_{i=1}^{n}[H(Y_{2,i}, Y_{3,i}, Y_{5,i}, ..., Y_{2K-1,i}|Y_2^{i-1}, Y_3^{i-1}, Y_5^{i-1}, ..., \\
& Y_{2K-1}^{i-1}, Y_4^n, M_{34}, M_A, X_4^n, X_2^i, X_3^i, X_5^i, ..., X_{2K-1}^i)] \\
& \overset{(a)}{=} \sum_{i=1}^{n}[H(Y_{2,i}|Y_2^{i-1}, Y_3^{i-1}, Y_5^{i-1}, ..., Y_{2K-1}^{i-1}, Y_4^n, M_{34}, M_A, \\
& X_4^n, X_2^i, X_3^i, X_5^i, ..., X_{2K-1}^i, X_6^i, X_8^i, ..., X_{2K}^i)] \\
& \leq \sum_{i=1}^{n}[H(S^{N-p}X_{1,i}|S^{N-q}X_{1,i})] \\
& = n[p-q]^+
\end{aligned}
$$

where in (a) we use a Markov chain that given $(M_{65}, Y_4^n, X_3^i, X_5^i, ..., X_{2K-1}^i)$ and the symmetric nature (channel gains in the cross links between transmitter 1 and the non-desired receivers are the same) of the model, we can construct X_6^i. Similarly, $X_8^i, ..., X_{2K}^i$ are constructed. Combining everything and considering the symmetric rate, completes the proof. ∎

Remark 1: We note that in the regime $2/3 < \alpha < 2$ (which contains the "moderately weak and strong interference regimes in the terminology of [8]), this capacity result is the same as that of two K-user ICs operating simultaneously in both directions, i.e. the same as two K-user ICs as in [8].The technique is standard and similar to the proof in [10], but with more messages and care to be taken because of the interaction.

Remark 2: For the case of $\alpha = 1$, the channel gains in the direct links and cross links are the same. This point has been shown to be discontinuous in the generalized degrees of freedom for the one-way K-user IC [8] (the value is $1/K$) because at this point all receivers receive exactly same signals for the linear deterministic model. However, if one considers time-varying channels which grow at the same rate but are not necessarily identical, the value of this point, which is known as the degrees of freedom, is shown to be $K/2$ [11], which is achieved by interference alignment. The same result has been shown for almost all (excluding a set of measure zero) constant channels in [12]. For the K-pair-user two-way IC, we show the *degrees of freedom* is K in [13] for both time-varying or constant channels.

B. *Constant gap result for Gaussian model*

We next derive an outer bound for the Gaussian interactive two-way K-user IC and show that, for certain "medium interference" regimes to be specified, this lies to within a constant gap of the outer bounds for two one-way, non-interactive K-user Gaussian ICs (for symmetric channels) of [9, Eq. (42)] which are identical to those of the 2-user IC [14], which in turn have been shown to lie within a constant gap of again, non-adaptive inner bounds for all channel gains outside a small outage set (we leave details of this "small outage set" to [9]). This means that, in the two regimes considered, "adaptation" or "interaction" may only provide a bounded gain.

Theorem 2: The symmetric sum-capacity of K-pair-user Gaussian two-way interference channel with interaction in the moderately weak interference ($\frac{2}{3}\log \mathtt{SNR} \leq \log \mathtt{INR} \leq \log \mathtt{SNR}$ or $\frac{2}{3} \leq \alpha \leq 1$) and the strong interference ($\log \mathtt{SNR} \leq \log \mathtt{INR} \leq 2\log \mathtt{SNR}$ or $1 \leq \alpha \leq 2$) regimes is within $\log(K)+\frac{K}{2}-1$ bits to the outer bound of two simultaneously operating one-way (non-interactive) Gaussian K-user interference channels, which in turn may be shown to be within a constant gap to non-adaptive inner bounds for all channel gains outside a small outage set, as done in [9].

Proof:

We first derive a new outer bound for our channel model and then show the gap result. Let $Z_{3,...,2K-1}$ be the vector of noises $Z_3, Z_5, ..., Z_{2K-1}$. Define $\bar{Z}_l = Z_l - Z_4, l = 6, 8, ..., 2K$. Let $\bar{Z}_{6,...,2K}$ denote $\bar{Z}_6, \bar{Z}_8, ..., \bar{Z}_{2K}$.

$$
\begin{aligned}
& n(R_{12}+R_{34}-\epsilon) \\
& \overset{(a)}{\leq} I(M_{34}; Y_4^n|M_A, Z_{3,...,2K-1}^n)+I(M_{12}; Y_2^n, Y_4^n|M_{34}, M_A, Z_{3,...,2K-1}^n) \\
& \overset{(b)}{\leq} I(M_{34}; Y_4^n, \bar{Z}_{6,...,2K}^n|M_A, Z_{3,...,2K-1}^n) \\
& + I(M_{12}; Y_2^n, Y_4^n, \bar{Z}_{6,...,2K}^n|M_{34}, M_A, Z_{3,...,2K-1}^n) \\
& = H(Y_4^n, \bar{Z}_{6,...,2K}^n|M_A, Z_{3,...,2K-1}^n) \\
& + H(Y_2^n|Y_4^n, \bar{Z}_{6,...,2K}^n, M_{34}, M_A, Z_{3,...,2K-1}^n) \\
& - H(Y_2^n, Y_4^n, \bar{Z}_{6,...,2K}^n|M_{12}, M_{34}, M_A, Z_{3,...,2K-1}^n)
\end{aligned}
$$

where (a) follows as all messages and noises are independent; (b) by adding the side information $\bar{Z}_{6,...,2K}^n$. We bound the three terms above respectively. For the first term:

$$
\begin{aligned}
& H(Y_4^n, \bar{Z}_{6,...,2K}^n|M_A, Z_{3,...,2K-1}^n) \leq H(Y_4^n)+H(\bar{Z}_{6,...,2K}^n) \\
& \leq H(g_{14}X_1^n+g_{34}X_3^n+...+g_{2K-1,4}X_{2K-1}^n+Z_4^n)+H(\bar{Z}_{6,...,2K}^n) \\
& \overset{(a)}{\leq} n\log 2\pi e(1+\mathtt{SNR}+(K-1)\mathtt{INR}+2(K-1)\sqrt{\mathtt{SNR}\times\mathtt{INR}} \\
& +(K-1)(K-2)\mathtt{INR})+n(K-2)\log(2\pi e)(2)
\end{aligned}
$$

where in (a) we have used the fact that Gaussians maximize entropy subject to power constraints and the symmetric channel model. Due to interaction, the inputs $X_l, l \in \{1, 3, ..., 2K-1\}$ may be correlated, and so we have upper bounded this term by assuming all the transmitters have the same power and they are maximally (fully) correlated.

The second term can be bounded as follows:

$$H(Y_2^n|Y_4^n,\bar{Z}_{6,...,2K}^n,M_{34},M_A,Z_{3,...,2K-1}^n)$$
$$\leq H(Y_2^n,Y_3^n,Y_5^n,...,Y_{2K-1}^n|Y_4^n,\bar{Z}_{6,...,2K}^n,M_{34},M_A,Z_{3,...,2K-1}^n)$$
$$=\sum_{i=1}^n[H(Y_{2,i},Y_{3,i},Y_{5,i},...,Y_{2K-1,i}|Y_2^{i-1},Y_3^{i-1},Y_5^{i-1},...,Y_{2K-1}^{i-1},$$
$$Y_4^n,\bar{Z}_{6,...,2K}^n,M_{34},M_A,X_2^i,X_3^i,X_5i,...,X_{2K-1}^i,X_4^n,Z_{3,...,2K-1}^n)]$$
$$\overset{(b)}{=}\sum_{i=1}^n[H(Y_{2,i},Y_{3,i},Y_{5,i},...,Y_{2K-1,i}|Y_2^{i-1},Y_3^{i-1},Y_5^{i-1},...,Y_{2K-1}^{i-1},$$
$$Y_4^n,\bar{Z}_{6,...,2K}^n,M_{34},M_A,X_2^i,X_3^i,X_5^i,...,X_{2K-1}^i,X_4^n,Z_{3,...,2K-1}^n,$$
$$X_{6,i},X_{8,i},...,X_{2K,i})]$$
$$\leq\sum_{i=1}^n[H(g_{12}X_{1,i}+Z_{2,i}|g_{14}X_{1,i}+Z_{4,i})]$$
$$=n\log 2\pi e\left(1+\frac{\texttt{SNR}}{1+\texttt{INR}}\right)$$

where in step (b) we construct $X_{6,i}$ in the conditioning because (1), $g_{14}X_1^{i-1}+Z_4^{i-1}$ can be decoded from Y_4^n since $X_3^i,X_5^i,...,X_{2K-1}^i$ are known; (2), $\bar{Z}_6^{i-1}$ is known so that $g_{14}X_1^{i-1}+Z_6^{i-1}$ can be constructed; and (3), we consider symmetric model, i.e. $g_{14}=g_{16}$. Therefore $g_{16}X_1^{i-1}+Z_6^{i-1}$ is known and then we can construct $X_{6,i}$. Similarly $X_{8,i},...,X_{2K,i}$ can be constructed.

Finally, the negative third term can be lower bounded as:

$$H(Y_2^n,Y_4^n,\bar{Z}_{6,...,2K}^n|M_{12},M_{34},M_A,Z_{3,...,2K-1}^n)$$
$$\geq H(Y_2^n,Y_4^n,\bar{Z}_{6,...,2K}^n|M_{12},M_{34},M_A,Z_{3,...,2K-1}^n,$$
$$X_1^n,X_3^n,X_5^n,...,X_{2K-1}^n)\geq H(Z_2^n,Z_4^n,\bar{Z}_{6,...,2K}^n)$$
$$=H(Z_2^n,Z_4^n,Z_6^n,...,Z_{2K}^n)=nK\log 2\pi e$$

Combining everything and considering the symmetric rate yields the following outer bound for the K-pair-user two-way Gaussian interference channel:

$$R_{sym}=\frac{R_{12}+R_{34}}{2}$$
$$\leq\frac{1}{2}\log(1+\texttt{SNR}+(K-1)^2\texttt{INR}$$
$$+2(K-1)\sqrt{\texttt{SNR}\times\texttt{INR}})+\frac{1}{2}\log\left(1+\frac{\texttt{SNR}}{1+\texttt{INR}}\right)+\frac{K-2}{2}$$

We show that our outer bound is to within a constant gap to existing non-adaptive outer bounds for K-user one-way Gaussian interference channel provided in [9, Eq. (42)].

1) The following non-adaptive bound is for moderately weak interference regime given by $\frac{2}{3}\log\texttt{SNR}\leq\log\texttt{INR}\leq\log\texttt{SNR}$ or $\frac{2}{3}\leq\alpha\leq 1$:

$$R_{sym1}\leq\frac{1}{2}\log(1+\texttt{SNR})+\frac{1}{2}\log(1+\frac{\texttt{SNR}}{1+\texttt{INR}})\qquad(6)$$

In this case, the gap may be bounded as

$$R_{sym}-R_{sym1}=\frac{1}{2}\log\frac{1+(K-1)^2\texttt{INR}+\texttt{SNR}+2(K-1)\sqrt{\texttt{SNR}\times\texttt{INR}}}{1+\texttt{SNR}}$$
$$+\frac{K-2}{2}\overset{\texttt{INR}<\texttt{SNR}}{\leq}\frac{1}{2}\log\left(1+\frac{(K^2-1)\texttt{SNR}}{1+\texttt{SNR}}\right)+\frac{K-2}{2}$$
$$\leq\frac{1}{2}\log(K^2)+\frac{K-2}{2}=\log K+\frac{K}{2}-1.$$

2) The following non-adaptive bound is for strong but not very strong interference regime given by $\log\texttt{SNR}\leq\log\texttt{INR}\leq 2\log\texttt{SNR}$ or $1\leq\alpha\leq 2$:

$$R_{sym2}\leq\frac{1}{2}\log(1+\texttt{SNR}+\texttt{INR})\qquad(7)$$

In this case, the gap may be bounded as

$$R_{sym}-R_{sym2}=$$
$$\frac{1}{2}\log\frac{1+(K-1)^2\texttt{INR}+\texttt{SNR}+2(K-1)\sqrt{\texttt{SNR}\times\texttt{INR}}}{1+\texttt{INR}}+\frac{K-2}{2}$$
$$\overset{\texttt{INR}>\texttt{SNR}}{\leq}\frac{1}{2}\log\left(1+\frac{(K^2-1)\texttt{INR}}{1+\texttt{INR}}\right)+\frac{K-2}{2}\leq\log K+\frac{K}{2}-1.$$

■

IV. CONCLUSION

We introduced the K-pair-user two-way Gaussian interference channel with interaction, derived new outer bounds, and demonstrated a capacity result for the linear deterministic model and a constant gap result for the Gaussian model, both in two "medium" interference regimes. These results indicate that, for these regimes and symmetric rates, interaction between users is useless or may only provide limited capacity gains, as is the case for one-way K-user ICs with feedback in these same regimes. Characterizing capacity and gap results for other interference regimes is an interesting topic for future work.

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