

CS 301 Languages and Automata, UIC
Fall 2012, Course Project
Due: Friday, December 7, 2012 in class

One of the goals of this course is to train you to be able to expand your education independently, given the tools you've learned here. For this reason, your project will consist of the following task.

Please independently read Chapter 8 on Space Complexity, up to and including PSPACE-completeness of the problem TBQF (Thm 8.9). This will introduce you to the following important concepts: Savitch's theorem, the class PSPACE, and a canonical problem complete for PSPACE.

Next, please answer the following questions.

1. [6 marks] Show that PSPACE is closed under the operations union, complementation, and star.
2. [8 marks] Let $L = \{\langle R, S \rangle \mid R \text{ and } S \text{ are equivalent regular expressions}\}$. Show that $L \in \text{PSPACE}$.
3. You must do one of the following. If you do both, the question on which you do less well will count as a bonus, up to a cap of an extra 3% on your final grade. For example, if you get 10 marks on (a), and 5 marks on (b), you will receive a bonus worth 1.5% on your final grade.

Describe in your own words (do not simply regurgitate the description in the text — you must convince us that you have understood the proof yourself, we will mark strictly. For example, for part (a) you should describe the reason behind each step in the procedure CANYIELD, the reason behind the design of M , and a brief explanation of the formal runtime. You should *not* explain every step in the text's proof; just the main steps which you feel are "significant". The goal here is to train you to ask the right questions when you read theorems and proofs; it's impossible to memorize every detail of every proof, but if you keep the main tricks and ideas in mind, it will help you solve problems in the future):

- (a) [10 marks] Why is Savitch's theorem perhaps surprising, given our study of P versus NP? What are the main ideas behind the proof of Savitch's theorem?
- (b) [10 marks] What is the difference between the boolean formulae we studied for the problem SAT and those for TBQF? What are the main ideas behind the proof of PSPACE-completeness for TBQF?