CS 301 Languages and Automata, UIC
Fall 2012, Assignment 6
Due: Friday, November 16, 2012 at start of discussion session

Unless otherwise noted, the alphabet for all questions below is assumed to be $\Sigma = \{0, 1\}$. This assignment will get you primarily to practice reductions. This is important, as the theory of NP-completeness we will see in Chapter 7 is based almost entirely on reductions.

1. [10 marks] This question is a warmup question.
   (a) Intuitively, what does the notation $A \leq B$ mean for problems A and B?
   (b) What is a mapping reduction $A \leq_m B$ from language A to language B? Give both a formal definition, and a brief intuitive explanation in your own words.
   (c) What is a computable function? Give both a formal definition, and a brief intuitive explanation in your own words.
   (d) Suppose $A \leq_m B$ for languages A and B. Please answer each of the following with a brief explanation.
      i. If B is decidable, is A decidable?
      ii. If A is undecidable, is B undecidable?
      iii. If B is undecidable, is A undecidable?
   (e) Show via a reduction from the halting problem that
   $$L = \{\langle M, w\rangle \mid M \text{ is a TM which does not halt on input } w\}$$
   is undecidable.
2. [8 marks] Let $T = \{\langle M\rangle \mid M \text{ is a TM that accepts } w^R \text{ whenever it accepts } w\}$. Show that T is undecidable. (Recall that w^R is the string w written in reverse, i.e. $011^R = 110$.)
3. (a) [8 marks] Consider the problem of determining whether a TM M on an input w ever attempts to move its head left when its head is on the left-most tape cell. Formulate this problem as a language and show that it is undecidable.
   (b) [5 marks] Consider the problem of determining whether a TM M on an input w ever attempts to move its head left at any point during its computation on w. Formulate this problem as a language and show that it is decidable.
4. (a) [4 marks] Recall Rice's Theorem from problem 5.28 in the text (also discussed in class). Show that both conditions in the statement of Rice's theorem (i.e. P is non-trivial, and P is a property of the TM's language) are necessary to prove that P is undecidable.
   (b) [6 marks] Use Rice's Theorem to prove the undecidability of the following languages.
      i. $L_1 = \{\langle M\rangle \mid M \text{ is a TM and } 1011 \in L(M)\}$.
      ii. $L_2 = \{\langle M\rangle \mid M \text{ is a TM and } L(M) = \Sigma^*\}$.

5. [6 marks] Consider the problem of determining whether a PDA accepts some string of the form $\{ww \mid w \in \{0,1\}^*\}$. Use the computation history method to show that this problem is undecidable. Hint: I suspect it will help to read pages 220-226 of the text in 3rd edition (pages 192-198 in 2nd edition), only part of which we had time to cover in class. Hint 2: Given an input $\langle M, w\rangle$ to A_{TM}, construct a PDA P which accepts a string of form HH if and only if H corresponds to a *valid* and *accepting* computation history for M on w.