CS 301 Languages and Automata, UIC
Fall 2012, Assignment 4
Due: Friday, October 12, 2012 at start of discussion session

Unless otherwise noted, the alphabet for all questions below is assumed to be $\Sigma = \{0, 1\}$.

1. [12 marks] This question develops a basic understanding of CFGs and parse trees. Consider the grammar G below.

$$E \rightarrow E + T \mid T \quad (1)$$
$$T \rightarrow T \times F \mid F \quad (2)$$
$$F \rightarrow (E) \mid 2 \quad (3)$$
$$(4)$$

   For each string below, give a parse tree and a derivation in G.

   (a) 2

   (b) $2 + 2 + 2$

   (c) $((2 + 2) \times (2))$

2. [10 marks] We have seen in class that the sets of both regular and context-free languages are closed under the union, concatenation, and star operations. We have also seen in A2 that the regular languages are closed under intersection and complement. In this question, you will investigate whether the latter also holds for context-free languages.

   (a) Use the languages $A = \{a^m b^n c^n \mid m, n \geq 0\}$ and $B = \{a^n b^n c^m \mid m, n \geq 0\}$ to show that the class of context-free languages is not closed under intersection. You may use the fact that the language $C = \{a^n b^n c^n \mid n \geq 0\}$ is not context-free.

   (b) Using part (a) above, show now that the set of context-free languages is not closed under complement.

3. [15 marks] This question develops your ability to design CFGs. For each of the following languages, give a CFG. Assume the alphabet is $\Sigma = \{0, 1\}$. For string $x = x_1 x_2 \cdots x_n$, we define $x^R := x_n \cdots x_2 x_1$, i.e. this operation reverses the order of the symbols in x.

   (a) $\{x \mid x$ starts and ends with the same symbol$\}$.

   (b) $\{x \mid x$ the length of x is odd$\}$.

   (c) $\left\{x \mid x^R\right.$, that is, x is a palindrome$\left.\right\}$.

   (d) $\emptyset$.

   (e) For this part, set $\Sigma = \{a, b, \$\}$. The language is then

$$\left\{x_1 \$ x_2 \$ \cdots \$ x_k \mid k \geq 1, \text{ each } x_i \in \{a, b\}^*, \text{ and for some } i \text{ and } j,\ x_i = x_j^R\right\}.$$

4. [15 marks] This question develops your ability to design PDAs. For parts (a), (b), (c), and (d) of question 3 above, give state diagrams of pushdown automata. For each automata, include a brief description of the idea behind its design.

5. [10 marks] This question forces you to practice the generic construction for mapping a CFG to a PDA. Specifically, for the grammar G from question 1, use the construction from Theorem 2.20 to construct an equivalent PDA P.

You are only required to do *one* of the following two questions. If you do both, the question on which you do less well will count as a bonus question.

6. [10 marks] Show that if G is a CFG in Chomsky normal form, then for any string $w \in L(G)$ of length $n \geq 1$, exactly $2n - 1$ steps are required for any derivation of w.

7. [10 marks] Let G be a CFG in Chomsky normal form that contains b variables. Show that if G generates some string with a derivation having at least 2^b steps, then $L(G)$ is infinite.