CS202 Fall 2012
Lecture 8 – 9/25
Probability

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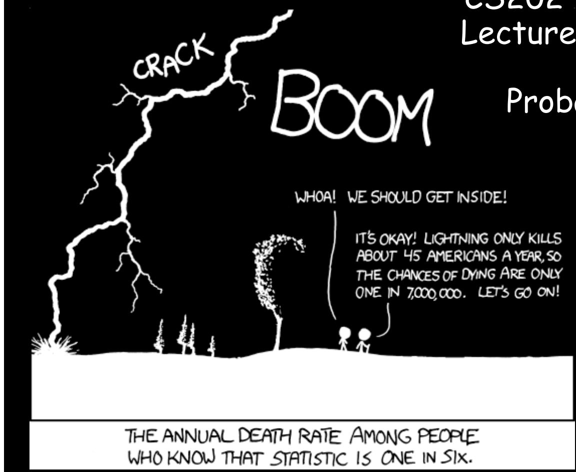
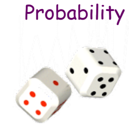


xkcd.com/795

Probability

Which is more likely:



a) Rolling an 8 when 2 dice are rolled?
b) Rolling an 8 when 3 dice are rolled?
c) No clue.

Probability

What is the probability of a total of 8 when 2 dice are rolled?

What is the size of the sample space? 36

How many rolls satisfy our condition of interest? 5

So the probability is 5/36.

Probability

What is the probability of a total of 8 when 3 dice are rolled?

What is the size of the sample space? 216

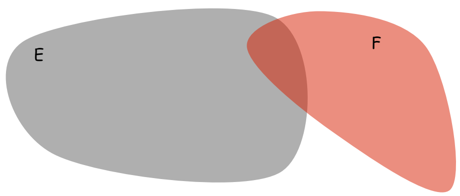
How many rolls satisfy our condition of interest? C(7,2)

So the probability is 21/216.

Conditional Probability

Let E and F be events with $Pr(F) > 0$. The conditional probability of E given F, denoted by $Pr(E|F)$ is defined to be:

$$Pr(E|F) = Pr(E \cap F)/Pr(F).$$



Conditional Probability

$$Pr(E|F) = Pr(E \cap F)/Pr(F).$$

A bit string of length 4 is generated at random so that each of the 16 bit strings is equally likely. What is the probability that it contains at least two consecutive 0s, given that its first bit is a 0?

$Pr(F) = 1/2$

$Pr(E \cap F)$? 0000 0001 0010 0011 0100

$Pr(E \cap F) = 5/16$ $Pr(E|F) = 5/8$

Independence

The events E and F are *independent* if and only if $Pr(E \cap F) = Pr(E) \times Pr(F)$.

Let E be the event that a family of n children has children of both sexes.
Lef F be the event that a family of n children has at most one boy.
Are E and F independent if

$n = 2$? No

Independence

The events E and F are *independent* if and only if $Pr(E \cap F) = Pr(E) \times Pr(F)$.

Let E be the event that a family of n children has children of both sexes.
Lef F be the event that a family of n children has at most one boy.
Are E and F independent if

$n = 3$? Yes

Independence

The events E and F are *independent* if and only if $Pr(E \cap F) = Pr(E) \times Pr(F)$.

Let E be the event that a family of n children has children of both sexes.
Lef F be the event that a family of n children has at most one boy.
Are E and F independent if

n = 4? No

Independence

The events E and F are *independent* if and only if $Pr(E \cap F) = Pr(E) \times Pr(F)$.

Let E be the event that a family of n children has children of both sexes.
Lef F be the event that a family of n children has at most one boy.
Are E and F independent if

n = 5? No

Independence

The events E and F are *independent* if and only if $Pr(E \cap F) = Pr(E) \times Pr(F)$.

Let E be the event that a family of n children has children of both sexes.
Lef F be the event that a family of n children has at most one boy.
Are E and F independent if

n = 2? No

n = 3? Yes

n = 4? No

n = 5? No

Bernoulli Trials

A coin is tossed 8 times. What is the probability of exactly 3 heads in the 8 tosses?

THHTTHTT is a tossing sequence…

How many ways of choosing 3 positions for the heads?

$C(8,3)$

What is the probability of a particular sequence?

$.5^8$

In general: The probability of exactly k successes in n independent Bernoulli trials with probability of success p, is

$C(n,k)p^k(1-p)^{n-k}$

Bernoulli Trials

A game of Jewel Quest is played 5 times. You clear the board 70% of the time. What is the probability that you win a majority of the 5 games?

Sanity check: What is the probability the the result is WWLLW?

$.7^3.3^2$ Assumes independent trials

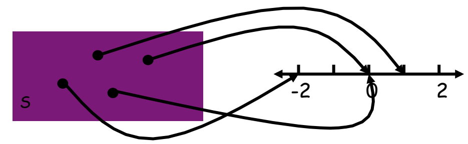
In general: The probability of exactly k successes in n independent Bernoulli trials with probability of success p, is

$C(n,k)p^k(1-p)^{n-k}$

$C(5,3)0.7^3 0.3^2 + C(5,4)0.7^4 0.3^1 + C(5,5)0.7^5 0.3^0$

Random Variables

For a given sample space S, a *random variable* is any real valued function on S.



Suppose our experiment is a roll of 2 dice. S is set of pairs.

- X = sum of two dice. X((2,3)) = 5
- Y = difference between two dice. Y((2,3)) = 1
- Z = max of two dice. Z((2,3)) = 3

Random Variables

A probability distribution on a r.v. X is just an allocation of the total probability, 1, over the possible values of X.

How many movies have you watched in the last week?

a) 0
b) 1
c) 2
d) 3
e) 4

Picture gives a probability distribution!

The chart gives likelihood that a randomly selected student watched each of the particular numbers of movies.

Random Variables

Example: Do you ever play the game Racko?

Suppose we are playing a game with cards labeled 1 to 20, and we draw 3 cards. We bet that the maximum card has value 17 or greater. What's the probability we win the bet?

Let r.v. X denote the maximum card value. The possible values for X are 3, 4, 5, …, 20.

i	3	4	5	6	7	8	9	…	20
Pr(X = i)	?	?	?	?	?	?	?		?

Filling in this box would be a pain. We look for a general formula.

Random Variables

X is value of the highest card among the 3 selected. 20 cards are labeled 1 through 20.

We want Pr(X = i), i = 3,…20.

Denominator first: How many ways are there to select the 3 cards? *C*(20,3)

a) 20
b) 6840
c) 60
d) 1140
e) I'm not telling.

How many choices are there that result in a max card whose value is i? *C*(i-1,2)

Pr(X = i) = *C*(i-1, 2) / *C*(20,3) These are the table values.

We win the bet if the max card, X is 17 or greater. What's the probability we win? Pr(X = 17) + Pr(X = 18) + Pr(X = 19) + Pr(X = 20) ≈0.51