

Q6.

Thm: If A and B are 1-equivalent, and if each implied pair is either

(a) a singleton

(b) (A, B) or

(c) (B, A)

then A and B are equivalent.

Proof:

We shall show that A and B are k-equivalent for any $k \geq 1$. Consider a k-length input sequence. Two cases arise:

1) The k-length i/p to (A,B) never takes the machine to a singleton state; thus the next states after every 1 length i/p in this k-length seq. is always (A,B) or (B,A). Since A and B are 1 equiv., each input in the k-length seq. produces an identical (1-length) o/p.

2) The k-length i/p to (A,B) takes the machine to a singleton state, say, C, for the first time after an i/p of length m, where $1 \leq m < k$. Then, for the initial m-length i/p the machine's current states are (A,B) or (B,A), and from the argument for case 1), the o/p's corresponding to the m-length i/p are identical if the machine starts from either A or B. Since after the m-length i/p, the machine will be in the same state C, irrespective of whether it started from A or B, the o/p of the machine from here on will be identical for any length i/p; thus it will be identical for the remaining (k-m)-length i/p. Thus such a k-length i/p to (A,B) also results in an identical o/p string.

Thus we have shown that any k-length i/p string applied to (A,B) produces identical o/p's. Hence A and B are equivalent states