ECE 465 Quiz 1: Solution

Prob. 1

Q: Consider the function $f(a,b,c,d) = \sum_m (2,3,6,7,9,13,14,15)$. Give the formal definition of (a) a *minterm,* (b) a *prime implicate,* (c) an *implicant,* and (d) *covering of one implicant by another*

AND

Illustrate the definition using a K-map, stating the product/sum term(s) corresponding to the examples of each of the above definitions in the K-map.

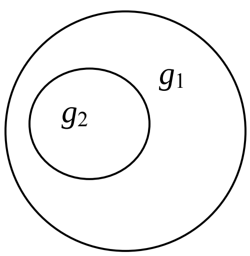
Soln:

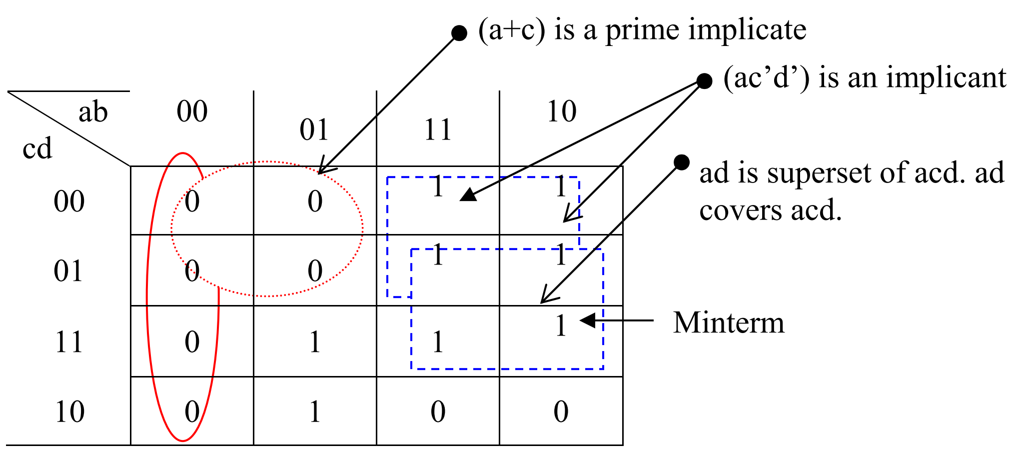
Minterm: a product of literals (complemented or uncomplemented form) containing each variable of the function exactly once.

Prime Implicate: On a K-map, a prime implicate is a group of adjacent maxterms that is not covered by a larger group of maxterm.

Implicant is a product term which covers minterms of a function. i.e. if g is an implicant of function f and if g=1 then f=1. So, an implicant covers one or more minterms of f.

Covering of one implicant by another if g_1 and g_2 are implicants of function f and g_1 covers g_2 then it means that g_1 is a superset of g_2.





Prob. 2

Q: Let *f* be a function with an implicant *g* and an implicate *h*. What can you say about the value of *f* when: (a) *g*=1; (b) *h*=1; (c) *g*=0; and (d) *h*=0?

Soln:

(a) *g*=1 ➔ *f*=1

(b) *h*=1 ➔ *f* depends on other implicates (if *h* is the only implicate, only then is f definitely 1)

(c) *g*=0 ➔ *f* depends on other implicants (if *g* is the only implicant, only then is f definitely 0)

(d) *h*=0 ➔ *f*=0

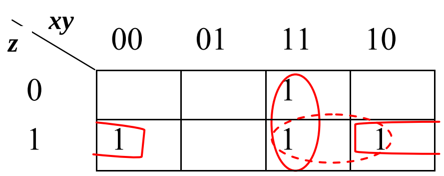
Prob. 3

Q: The consensus theorem is stated as:

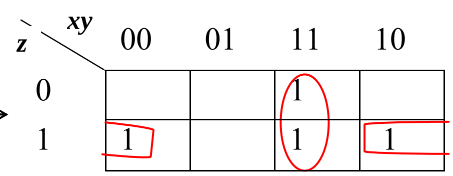
$$xy + \bar{y}z + xz = xy + \bar{y}z$$

Prove the consensus theorem using a K-map. Besides showing the necessary prime implicants grouping in the K-map, also state clearly in words how the K-map with the above prime implicants proves the theorem.

Soln:



All 3 Prime Implicants (PIs) xy, $\bar{y}z$, xz. The consensus xz of the other 2 PIs is shown with a dashed oval.



The consensus xz is not needed in the minimized expression as its minterms (MTs) are covered by the 2 PIs (xy, $\bar{y}z$) of which it is a consensus

Prob. 4

Q: (a) Consider a 3-bit number *X*. Provide a TT for the output number $Z = int(\sqrt{X})$, where *int*(*y*), is the integer portion of a real number *y*. Note that *Z* can be a multi-bit number.

(b) You need to give a 2-level NOR-gate circuit for *Z*. For this purpose either obtain a minimized SOP or minimized POS expression for each bit of *Z* using a K-map (whichever you think is more convenient for an implementation using only NOR gates) and then obtain the NOR-gate circuit using this minimized expression.

Assume that NOR gates with any number of inputs that you need are available, and that all variables and their complemented forms (i.e., all *literals*) are available.

Soln:

X			*Z*	
x_2	x_1	x_0	z_1	z_0
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	0	1
1	0	0	1	0
1	0	1	1	0
1	1	0	1	0
1	1	1	1	0

Generate POS, $z_0 = \prod_M (0,4,5,6,7) = x_2' \, (x_1 + x_0)$ after simplification using K-map

$z_1 = \prod_M (0,1,2,3) = x_2$