Fuzzy Signals in Control Loops

Rainer Palm
Siemens AG, R&D
Munich, Germany

Abstract— **The paper deals with fuzzy signals in control loops. With respect to specific effects coming up with the use of sensory information like noise or spatial distribution of a signal it is of interest how the control loop behaves in the presence of fuzzy signals. In this paper instationary fuzzy sets, especially time variant membership functions and their derivatives, are described. On this basis the gain scheduling control scheme according to Takagi/Sugeno is discussed. It is shown that, also in the case of fuzzy signals in the control loop, global stability can be proven.**

Keywords: Fuzzy control, fuzzy inputs, noisy signals, Takagi/Sugeno controller

I. Introduction

Process signals which appear within the control loop and normally treated as crisp values are often found to be disturbed by different kinds of noise so that they have to be processed in a special way (e.g. filtering, regression analysis etc.) in order to obtain satisfactory control results [Schwartz 59]. Noisy signals are more or less of ambiguous quality because the level of confidence in a single measurement at a certain time event strongly depends on the dispersion of the signal.

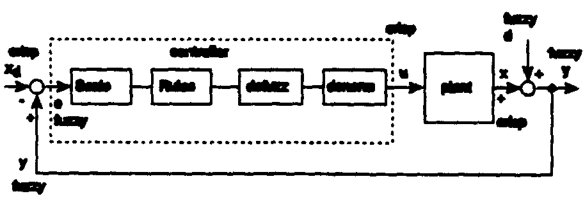

Figure 1: Control loop with fuzzy signals

Another type of ambiguity appears when, instead of a single sensor, a sensor array is employed whose individual subsensors provide different information (e.g. different intensities of radiation). The output of a sensor array can be processed subsensor by subsensor. A more sophisticated way is to gather all sensor data to a distribution that considers the subsensors and their individual level of information as a whole.
The difference between the two types of signals is that the first one is represented by a time series of single values whereas the second type provides a spatial distribution at a specific time event.
The two types of signals can be treated in a unified way if one derives a probability distribution from the noisy signal.
The question is how such ambiguous signals can be treated in a control loop.

The common way to deal with such a signal, while using conventional controllers, is to compute the average of its distribution and provide the controller with this value. However, in this case the information about standard deviation and the higher moments gets lost. The use of fuzzy controllers becomes therefore advantageous where the distribution, either coming from spatial information or from probability considerations, is interpreted as a membership function of the fuzzy set "around $\bar{x}$" if $\bar{x}$ is the mean value of the distribution.
The scope of pure fuzzy systems including fuzzy signals has been extensively studied by [Tong 80, Gupta 86, Pedrycz 92]. Nevertheless, one also should pay attention to the mixed case where some signals are crisp and some are fuzzy. This is the case when the objective $\mathbf{x_d}$ is crisp and the output $\mathbf{y}$, fed back via sensors, is fuzzy. Then, the error signal $\mathbf{e}$ is also fuzzy (see fig. 1). Error $\mathbf{e}$ is fed to the input of a fuzzy controller without any fuzzification block because the input signal $\mathbf{e}$ is already a fuzzy value. The output u of the controller is crisp since the system to be controlled requires crisp inputs. The crisp state $\mathbf{x}$ of the system is measured by means of sensors providing the fuzzy output vector $\mathbf{y}$.

The transformation of the noisy or spatial distributed signal into a fuzzy set is done as follows:

- Construction of a histogram from a probabilistic or spatial distribution of the signal to be considered
- Transformation of the histogram into a fuzzy set via normalization with respect to the maximum value of the histogram
- Feedback of the fuzzy signal to the controller input.

The major reason why this option is worth investigating is to take into account as much information describing the signal measured as possible. This information is used while computing the controller output. It includes the confidence in a measurement represented by the standard deviation of the distribution, the degree of deformation and asymmetry according to a Gaussian distribution represented by the higher moments of the distribution and the occurance of more than one peak in the distribution. Normally, fuzzy sets are characterized by stationary and time invariant membership functions. However, in the context of fuzzy input signals the problem of time variant fuzzy sets arises. Therefore, some operations with regard to instationary fuzzy sets are defined especially the differentiation of a fuzzy set with respect to time.

Although some methods exist to prove stability of fuzzy controlled systems [de Glas 84, Aracil 89, Tanaka 92, Tanaka 93, Palm 92] all of these methods deal with crisp signals throughout the control loop. Therefore, it is of interest to find out corresponding methods for investigating stability and robustness of fuzzy controlled systems in the case of fuzzy signals at the input of the controller.

In [Palm 94] these points have been discussed for sliding mode control (SMC) [Utkin 77] and related control strategies i.e. SMC boundary layer [Slotine 85], and sliding mode fuzzy control (SMFC) [Kawaji 91, Palm 92, Hwang 92]. In [Yager 94, Mouzouris 94, Galichet 94] noisy inputs have also been discussed but not from an explicit control point of view.

Because of their hybrid quality, Takagi/Sugeno controllers play more and more an important role in fuzzy control [Takagi 85]. A last topic is therefore devoted to processing of fuzzy inputs within that type of controller. In this context it will be shown that, although fuzzy signals are present, global stability can be proven.

II. INSTATIONARY FUZZY SETS

Fuzzy sets with time variant parameters Normally, fuzzy sets are considered to be fixed in time and therefore stationary sets. If, however, some parameters of a fuzzy set are changing with time one has to call this type of fuzzy sets *instationary*. Let, for example, a fuzzy set $X(t)$ be described by a bell-shaped membership function $\mu_X(x(t))$ similar to a Gaussian probability distribution.

$$\forall t \qquad \mu_X(x(t)) = e^{-\frac{(x-\bar{x}(t))^2}{2\cdot\sigma(t)^2}} \tag{1}$$

where

$\bar{x}(t)$ - time variable mean
and
$\sigma(t)$ - time variable standard deviation (width).

Similar to a probability distribution we characterize the width of the membership function by a scaled deviation $\sigma(t)$. The fuzzy set is normal which means $\mu_X(x(t) = \bar{x}(t)) = 1$. Since $x(t)$ is a function of time the fuzzy set X moves along its universe of dicourse according to the velocity $\dot{\bar{x}}(t)$ of the mean $\bar{x}(t)$ and the velocity $\dot{\sigma}(t)$ of the deviation $\sigma(t)$. Thus, the dynamics of the membership function only depends on the two parameters $\bar{x}(t)$ and $\sigma(t)$. The representation of a time variable fuzzy set and its derivatives with respect to time by a finite number of parameters (in our case $\bar{x}(t)$ and $\sigma(t)$) is very useful to bridge some gaps between conventional and fuzzy system theory.

On the other hand, the representation of a time variable fuzzy set $X(t)$ in terms of its parameters is not a fuzzy set. The question is how the velocity of a given time variant fuzzy set in terms of a fuzzy set looks like? This includes the problem of how derivatives of a fuzzy set with respect to time are defined.

Differentiation of a fuzzy set with respect to time The proposed definition by [Dubois 80, Zimmermann 91] of the differentiation of a fuzzy set does not satisfy the problems arising for dynamical fuzzy sets with time variable parameters. Therefore, a different definition of the derivative of a fuzzy set with respect to time has been proposed [Palm 94]:

The differentiation of a crisp function $x(t)$ with respect to time is defined by

$$\dot{x} = lim_{\Delta t\to 0}\frac{x(t+\Delta t)-x(t)}{\Delta t}.$$

A regarding operation with respect to a fuzzy set can be achieved as follows Consider a fixed pair $\langle\mu_X(x^i(t)), x^i(t)\rangle$. The behavior of a fixed pair $\langle\mu_X(x^i(t)), x^i(t)\rangle$ with respect to time is based on the following condition:

$$\forall\Delta t \qquad x^i(t+\Delta t) = x^i(t) + \Delta x^i(t)$$

and

$$\mu_X(x^i(t+\Delta t)) = \mu_X(x^i(t)).$$

The fuzzy set $\dot{X}(t)$ is then defined as

$$\forall i \qquad \mu_{\dot{X}}(\dot{x}^i(t)) = max_k\{\mu_X(x^k(t))\} \tag{2}$$

where

$$\forall k \qquad \dot{x}^k(t) = \dot{x}^i(t).$$

This means, in the case of several points $x^k(t)$ with the same velocity $\dot{x}^i(t)$ but different degrees of membership $\mu_X(x^k)$ we choose the maximum degree of membership $max_k\{\mu_X(x^k(t))\}$ for $\dot{x}^i(t)$. This is justified because the fuzzy set $\dot{X}$ should be a normal set like $X(t)$.

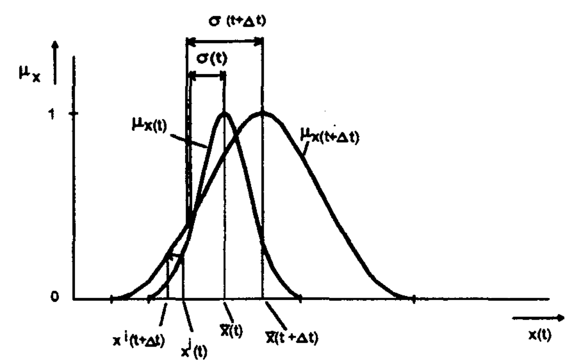

Figure 2: Motion of an instationary bell-shaped membership function along the x-axis of the universe of discourse

Let us now apply definition (2) to a bell-shaped membership function (see fig. (2)). Let $\mu_X(x^i(t))$ and $\mu_X(x^i(t+\Delta t))$ the membership functions for point x^i at time t and $t+\Delta t$, respectively:

$$\mu_{x^i(t)} = e^{-\frac{(x^i(t)-\bar{x}(t))^2}{2\cdot\sigma(t)^2}}; \quad \mu_{x^i(t+\Delta t)} = e^{-\frac{(x^i(t+\Delta t)-\bar{x}(t+\Delta t))^2}{2\cdot\sigma(t+\Delta t)^2}} \tag{3}$$

where

$$\mu_X(x^i(t)) = \mu_X(x^i(t+\Delta t)). \tag{4}$$

From eqs.(3) and (4) follows

$$\frac{x^i(t)-\bar{x}(t)}{\sigma(t)} = \frac{x^i(t+\Delta t)-\bar{x}(t+\Delta t)}{\sigma(t+\Delta t)}.$$

With the linear approximations

$$\begin{aligned} x^i(t+\Delta t) &\approx x^i(t) + \dot{x}^i\cdot\Delta t \\ \bar{x}(t+\Delta t) &\approx \bar{x}(t) + \dot{\bar{x}}\cdot\Delta t \\ \sigma(t+\Delta t) &\approx \sigma(t) + \dot{\sigma}\cdot\Delta t \end{aligned} \tag{5}$$

one obtains the velocity

$$\dot{x}^i(t) = \dot{\bar{x}}(t) + (x^i(t) - \bar{x}(t)) \cdot \frac{\dot{\sigma}(t)}{\sigma(t)}. \tag{6}$$

According to definition (2) the corresponding membership function for $\dot{x}^i(t)$ can be obtained by

1. For $\dot{\sigma} = 0$ one obtains $\forall i \quad \dot{x}^i(t) = \dot{\bar{x}}(t)$
   Since $\mu_{\dot{X}}(\dot{\bar{x}}(t)) = 1$ we obtain, according to our definition with respect to $\dot{X}$,

   $$\forall i \qquad \mu_{\dot{X}}(\dot{x}^i(t)) = 1$$

2. For $\dot{\sigma} \neq 0$ one obtains $\forall i \quad \mu_{\dot{X}}(\dot{x}^i(t)) = \mu_X(x^i(t))$. If $\forall i \quad \dot{x}^i(t) = \dot{\bar{x}}(t)$, as a special case, we obtain $\forall i \quad \mu_{\dot{X}}(\dot{x}^i(t)) = \mu_{\dot{X}}(\dot{\bar{x}}(t)) = \mu_X(\bar{x}(t))$

However, in practice mapping $\mu_X(x(t)) \rightarrow \mu_X(x(t + \Delta t))$ is complicated since the fuzzy sets measured are often not normal and even non-konvex. Therefore, a procedure of dealing with measured fuzzy sets is proposed which simplifies both the processing of the fuzzy set and the computation of its velocity.

Approximation of measured fuzzy sets with piecewise bell-shaped functions

Dealing with an instationary fuzzy signal and the rather complicated method of calculating the fuzzy set of its velocity out of the measurements requires a simplification of the whole procedure. This can be achieved through approximation of the signal distribution measured by means of bell-shaped functions. By means of this method one is able to approximate unimodal but asymmetrical distributions. With the approximation at time t and $t + \Delta t$ the fuzzy set of the velocity can be obtained easily.

In order to deal with this problem we start with the fuzzy set of the velocity of an instationary bell-shaped fuzzy set whose parameters are mean $\bar{x}(t)$ and standard deviation $\sigma(t)$. From the first formula of eqs.(3) and from eq.(6) we directly obtain the corresponding fuzzy set of the velocity

$$\mu_{\dot{X}}(\dot{x}^i(t)) = e^{-\frac{(\dot{x}^i(t) - \dot{\bar{x}}(t))^2}{2 \cdot \dot{\sigma}^2(t)}} \tag{7}$$

For a process that is assumed to be approximately Gaussian distributed its bell-shaped membership function is computed by the estimation of mean $\bar{x}(t)$ and standard deviation $\sigma(t)$. Knowing the time derivatives $\dot{\bar{x}}(t)$ and $\dot{\sigma}(t)$ it is therefore easy to compute the bell-shaped membership function of its velocity as well (see eq.(7)). For a lopsided (asymmetrical) but unimodal distribution a similar procedure holds:

It is assumed that an asymmetrical membership function $\mu_X(x^i(t))$ with $x^i \in [x_0, x_1]$ can be approximated by the left and right half of two symmetrical bell-shaped functions with the same mean $\bar{x}_L(t) = \bar{x}_R(t) = x_{max(\mu_X(x^i(t)))}$ but different standard deviations $\sigma_L(t) \neq \sigma_R(t)$. The left and right standard deviation, respectively, is obtained by dividing the original membership function measured in two halves at $x_{max(\mu_X(x^i(t)))}$ building up two symmetrical membership functions. From these two functions the standard deviations $\sigma_L(t)$ and $\sigma_R(t)$ are estimated resulting in two different membership functions μ_{X_L} and μ_{X_R} put together at $x_{max(\mu_X(x^i(t)))}$:

$$\begin{aligned} \mu_{X_L}(x^i(t)) &= e^{-\frac{(x^i(t) - \bar{x}_L(t))^2}{2 \cdot \sigma_L^2(t)}} \qquad \text{for} \qquad x_0 \leq x^i \leq \bar{x}_L(t) \\ \mu_{X_R}(x^i(t)) &= e^{-\frac{(x^i(t) - \bar{x}_R(t))^2}{2 \cdot \sigma_R^2(t)}} \qquad \text{for} \qquad \bar{x}_R(t) \leq x^i \leq x_1 . \end{aligned} \tag{8}$$

From the behavior of these approximations with respect to time the parameters $\dot{\bar{X}}_L(t) = \dot{\bar{X}}_L(t)$, $\dot{\sigma}_L(t)$, and $\dot{\sigma}_R(t)$ are to be computed from which we obtain $\mu_{\dot{X}}(x^i(t))$ with $\dot{x}^i \in [\dot{x}_0, \dot{x}_1]$

$$\begin{aligned} \mu_{\dot{X}_L}(\dot{x}^i(t)) &= e^{-\frac{(\dot{x}^i(t) - \dot{\bar{x}}_L(t))^2}{2 \cdot \dot{\sigma}_L^2(t)}} \qquad \text{for} \qquad \dot{x}_0 \leq \dot{x}^i \leq \dot{\bar{x}}_L(t) \\ \mu_{\dot{X}_R}(\dot{x}^i(t)) &= e^{-\frac{(\dot{x}^i(t) - \dot{\bar{x}}_R(t))^2}{2 \cdot \dot{\sigma}_R^2(t)}} \qquad \text{for} \qquad \dot{\bar{x}}_R(t) \leq \dot{x}^i \leq \dot{x}_1 . \end{aligned} \tag{9}$$

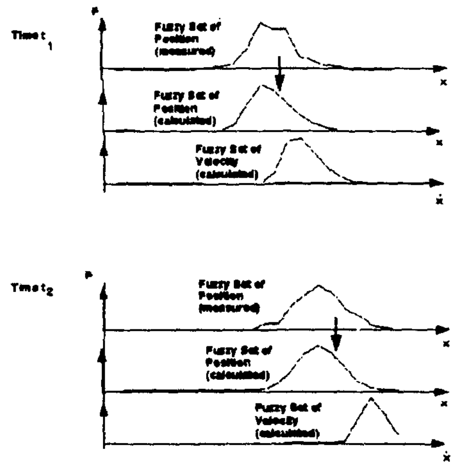

Figure 3: Approximation of measured membership functions and computation of their velocity

Figure (3) shows an example in which a zero mean Gaussian process y with standard deviation $\sigma_y = 1$ is multiplicatively and additively affected by sinusoidal functions. The resulting stochastical process x consists therefore of a pure random process y and some non-stochastical signal components:

$$x(t) = 0.5 \cdot \sin(0.8 \cdot t + 0.5) \cdot y + 4 \cdot sin(0.4 \cdot t).$$

The sample time for measuring $x(t)$ is $dt = 0.01s$. In order to obtain the distribution $p(x)$ of x for a specific time event t_i 200 $x(t)$ values are measured to fill in a histogram of 22 classes which corresponds to a time period of $2s$. After gathering the distribution $p(x)_{t=t_i}$ each value of $p(x)$ is normalized with respect to the maximum $p(x)_{max}$ of $p(x)$. The result is a fuzzy set $\mu_x(t_i)$. On the other hand, $p(x)_{t=t_i}$ provides mean and standard deviation of the distributions at the left and right hand side of the maximum value of $\mu_x(t_i)$ at time t_i. From these parameters two approximated bell-shaped membership functions for the left and right part are straight forward obtained. The next action is to perform the same steps for $t = t_{i+1}$. From this information the velocities of the mean and the individual standard deviations have been calculated. Finally, according to eqs.(9) the membership function $\mu_{\dot{x}}$ of the velocity of the fuzzy position has been calculated.

It should be noted that the information about the signs of the velocities of the standard deviations σ_L and σ_R of the original fuzzy set is not preserved in the fuzzy set of the velocity. This is in contrast to the velocity of the maximum value of the original fuzzy set whose sign and absolute value characterize the maximum of the resulting fuzzy velocity.

III. Fuzzy inputs and the Takagi/Sugeno controller

Rules for system and controller In [Palm 94, Driankov 94] it has been described how a fuzzy controller of Mamdani type deals with fuzzy values. On the other hand, because of their hybrid nature, Takagi/Sugeno controllers play more and more an essential role in fuzzy control. It is therefore of interest how a similar procedure could be performed in the case where both the system and the controller is described by a formulation according to Takagi/Sugeno [Takagi 85, Tanaka 92, Tanaka 93]. Let, therefore, the nonlinear state space be partitioned into m quasi linear regions $REG_1 \dots REG_m$ in which linear state equations hold [Driankov 93]. Then, the system can be formulated by the following set of rules

$$R_i^s: \quad IF \quad \mathbf{x_i} = \mathbf{X_i} \quad THEN \quad \dot{\mathbf{x}}_\mathbf{i} = \mathbf{A_i} \cdot \mathbf{x} + \mathbf{b_i} \cdot u \tag{10}$$

with

$\mathbf{x} = (x, \dot{x}, \dots, x^{(n-1)})^T$ - vector of crisp states
$\mathbf{X_i} = (X_1^i, X_2^i, \dots, X_n^i)^T$ - vector of linguistic variables for the crisp states ,
$\mathbf{A_i}$ - system matrix of region REG_i
$\mathbf{b_i}$ - control vector of region REG_i
$i = 1 \dots m$.

In these rules both the states and the control are crisp. However, the corresponding observation equation

$$\mathbf{y} = \mathbf{x} + \tilde{\mathbf{d}} \tag{11}$$

with the stochastic disturbance $\tilde{\mathbf{d}}$ provides a stochastically disturbed output $\mathbf{y}$. Both $\tilde{\mathbf{d}}$ and $\mathbf{y}$ are interpreted as fuzzy variables. On the basis of fuzzy output $\mathbf{y}$ error e is defined by

$$\mathbf{e} = \mathbf{y} - \mathbf{x_d} \tag{12}$$

with

$\mathbf{x_d}$ - crisp desired state vector
$\mathbf{e} = (e, \dot{e}, \dots, e^{(n-1)})^T$ - fuzzy error vector.

In contrast to commonly used Takagi/Sugeno control rules the corresponding control rules for fuzzy values of $\mathbf{e}$ are slightly different. In order to obtain a crisp control output u from a set of control rules by avoiding costly fuzzy arithmetic operations we defuzzify $\mathbf{e}$ by any appropriate defuzzification method (e.g. center of gravity):

$$\mathbf{e}^* = defuzz(\mathbf{e}) = \mathbf{x} - \mathbf{x_d} + defuzz(\tilde{\mathbf{d}}). \tag{13}$$

Then, we apply the fuzzy variable $\mathbf{e}$ to the premise part and the defuzzified value $\mathbf{e}^*$ to the consequence part of the rule. The result is that the information about shape and location of the fuzzy sets of $\mathbf{e}$ in relation to predefined linguistic variables $\mathbf{E_j}$ is preserved whereas the calculations in the consequence part only deal with crisp values $\mathbf{e}^*$.

Hence, the control rule for region REG_j yields

$$R_j^c: \quad IF \quad \mathbf{e} = \mathbf{E_j} \quad THEN \quad u_j = \mathbf{c_j^T} \cdot \mathbf{e}^* \tag{14}$$

with

$\mathbf{e}^* = (e^*, \dot{e}^*, \dots, e^{*(n-1)})^T$ - vector of defuzzified errors
$\mathbf{E_j} = (E_1^j, E_2^j, \dots, E_n^j)^T$ - vector of linguistic variables for the fuzzy errors
$\mathbf{c_j} = (c_1^j, c_2^j, \dots, c_n^j)^T$ - vector of control parameters
$j = 1 \dots m$

The rules R_i^s of (10) lead to the final state equation

$$\dot{\mathbf{x}} = \frac{1}{\sum_{i=1}^{m} w_i} \cdot \sum_{i=1}^{m} w_i \cdot (\mathbf{A_i} \cdot \mathbf{x} + \mathbf{b_i} \cdot u) \tag{15}$$

where

w_i - weight of rule R_i^s,
$\forall i = 1 \dots m \quad w_i \geq 0, \quad \sum_{i=1}^{m} w_i > 0.$

Furthermore, the control rules R_j^c of (14) lead to the overall control value

$$u = \frac{\sum_{j=1}^{m} v_j \cdot \mathbf{c_j^T}}{\sum_{j=1}^{m} v_j} \cdot \mathbf{e}^* \tag{16}$$

where

v_j - weight of rule R_j^c
$\forall j = 1 \dots m \quad v_j \geq 0, \quad \sum_{j=1}^{m} v_j > 0.$

Substitution of u in (15) by (16) finally yields the state equation of the whole system with fuzzy input signals:

$$\dot{\mathbf{x}} = \frac{1}{\sum_{i,j=1}^{m} w_i \cdot v_j} \cdot \sum_{i,j=1}^{m} w_i \cdot v_j \cdot (\mathbf{A_i} \cdot \mathbf{x} + \mathbf{b_i} \cdot \mathbf{c_j^T} \cdot \mathbf{e}^*) \tag{17}$$

It has to be emphasized that, in general, in contrast to crisp inputs

$$w_i \neq v_i. \tag{18}$$

Stability In order to check the system's stability eq. (15) is slightly changed into

$$\dot{\mathbf{x}} = \frac{1}{\sum_{i,j=1}^{m} w_i \cdot v_j^*} \cdot \sum_{i,j=1}^{m} w_i \cdot v_j^* \cdot (\mathbf{A_i} \cdot \mathbf{x} + \mathbf{b_i} \cdot \mathbf{c_j^T} \cdot \mathbf{e}^*) \tag{19}$$

with

$\forall j = 1 \dots m \quad v_j^* \geq 0, \quad \sum_{j=1}^{m} v_j^* > 0$
$w_i \neq v_i^*$ for $defuzz(\tilde{\mathbf{d}} \neq \mathbf{0})$

where v_j^* is the degree of membership which would have been obtained if one had used the defuzzified value $\mathbf{e}^* = defuzz(\mathbf{e})$ in the premise of rule (14). Stability is proved for the homogeneous part of eq. (19)

$$\dot{\mathbf{x}} = \frac{1}{\sum_{i,j=1}^{m} w_i \cdot v_j^*} \cdot \sum_{i,j=1}^{m} w_i \cdot v_j^* \cdot \tilde{\mathbf{A}}_\mathbf{ij} \cdot \mathbf{x} \tag{20}$$

with

$$\tilde{\mathbf{A}}_\mathbf{ij} = \mathbf{A_i} + \mathbf{b_i} \cdot \mathbf{c_j^T}.$$

According to [Driankov 93] system (20) is asymptotically stable in the large (*strong stability condition*) if there exists a common positive definite matrix $\mathbf{P}$ such that

$$\forall i,j = 1 \dots m \qquad \tilde{\mathbf{A}}_{\mathbf{ij}}^{\mathbf{T}}\mathbf{P} + \mathbf{P}\tilde{\mathbf{A}}_{\mathbf{ij}} < \mathbf{0}. \tag{21}$$

This is a remarkable result because (21) does not depend on any rule weight w_i and v_j^*.

The same result would have been obtained for the original degrees of membership v_j. In this case equation (20) is changed into

$$\dot{\mathbf{x}} = \frac{1}{\sum_{i,j=1}^{m} w_i \cdot v_j} \cdot \sum_{i,j=1}^{m} w_i \cdot v_j \cdot \tilde{\mathbf{A}}_{\mathbf{ij}} \cdot \mathbf{x} \tag{22}$$

from where (21) follows as well.

However, eq.(21) does ensure global stability but it is not a neccessary condition for asymptotical stability in the large. In other words, system (17) may be asymptotical stable in the large even if (21) is not satisfied. A weaker stability condition is therefore to be formulated where the rule weights w_i and v_j^* are involved:

$$\sum_{i,j=1}^{m} w_i \cdot v_j^* \cdot (\tilde{\mathbf{A}}_{\mathbf{ij}}^{\mathbf{T}}\mathbf{P} + \mathbf{P}\tilde{\mathbf{A}}_{\mathbf{ij}}) < \mathbf{0} \tag{23}$$

According to [Tanaka 93] condition (23) is called a "weak" condition for global stability. The term "weak" means that both the degrees of membership w_i and v_j^* are taken into account. This means that for every point in the state space the system (20) could be tested on stability if matrix $\tilde{\mathbf{A}}_{\mathbf{ij}}$ and the weights w_i, v_j^* are known.

For proving weak stability, however, we have to take into account the *system output uncertainty*. This leads to a "weak stability with uncertainty" which is a formulation of global stability in the sense that the consideration of output uncertainty provides information on the largest admissible uncertainty so that weak stability is still achieved.

Therefore, *weak stability with uncertainty* is achieved by changing (23) into

$$\sum_{i,j=1}^{m} w_i \cdot v_j \cdot (\tilde{\mathbf{A}}_{\mathbf{ij}}^{\mathbf{T}}\mathbf{P} + \mathbf{P}\tilde{\mathbf{A}}_{\mathbf{ij}}) < 0 \tag{24}$$

with

$$\begin{array}{lll} v_j = v_j^* + \Delta v_j & & \\ v_i^* + v_{i-1}^* = 1; & v_i^* + v_{i+1}^* = 1 & \\ w_i + w_{i-1} = 1; & w_i + w_{i+1} = 1 & \\ \forall \Delta v_i \neq 0 & v_i + v_{i-1} \neq 1; & v_i + v_{i+1} \neq 1 \\ -1 \leq \Delta v_i \leq 1; & 0 \leq v_i \leq 1 & \end{array}$$

In order to prove eq.(24) for a general case one has to compute

$$\mathbf{Q}_{\mathbf{ij}} = \tilde{\mathbf{A}}_{\mathbf{ij}}^{\mathbf{T}}\mathbf{P} + \mathbf{P}\tilde{\mathbf{A}}_{\mathbf{ij}}$$

with

$$\mathbf{Q}_{\mathbf{ij}} = \begin{pmatrix} q_{11} & q_{12} & \cdots & q_{1n} \\ q_{21} & q_{22} & \cdots & q_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ q_{n1} & q_{n2} & \cdots & q_{nn} \end{pmatrix}_{ij}.$$

From eq.(24) we then immediately obtain

$$\begin{pmatrix} \sum_{i,j=1}^{m} w_i v_j q_{11} & \sum_{i,j=1}^{m} w_i v_j q_{12} & \cdots & \sum_{i,j=1}^{m} w_i v_j q_{1n} \\ \sum_{i,j=1}^{m} w_i v_j q_{21} & \sum_{i,j=1}^{m} w_i v_j q_{22} & \cdots & \sum_{i,j=1}^{m} w_i v_j q_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \sum_{i,j=1}^{m} w_i v_j q_{n1} & \sum_{i,j=1}^{m} w_i v_j q_{n2} & \cdots & \sum_{i,j=1}^{m} w_i v_j q_{nn} \end{pmatrix}_{ij} < 0.$$

This means, according to Sylvester's theorem, that the following sub-conditions must hold:

$$\begin{gathered} \sum_{i,j=1}^{m} w_i v_j q_{11,j} < 0 \\ \begin{vmatrix} \sum_{i,j=1}^{m} w_i v_j q_{11} & \sum_{i,j=1}^{m} w_i v_j q_{12} \\ \sum_{i,j=1}^{m} w_i v_j q_{21} & \sum_{i,j=1}^{m} w_i v_j q_{22} \end{vmatrix}_{ij} < 0 \\ \vdots \\ \begin{vmatrix} \sum_{i,j=1}^{m} w_i v_j q_{11} & \cdots & \sum_{i,j=1}^{m} w_i v_j q_{1n} \\ \vdots & \ddots & \vdots \\ \sum_{i,j=1}^{m} w_i v_j q_{n1} & \cdots & \sum_{i,j=1}^{m} w_i v_j q_{nn} \end{vmatrix}_{ij} < 0. \end{gathered} \tag{25}$$

However, even for a 2nd order system and only two quasi linear regions considered ($i,j = 1 \dots 2$) it is not easy to prove weak stability with uncertainty [Tanaka 93]. Therefore, for a system of higher order and more than two quasi linear regions considered a computation of the *admissible measurement uncertainty* becomes inefficient and the *strong stability condition* (21) should be prefered.

IV. Conclusion

The paper dealt with fuzzy signals at the controller input where the fuzzy input originates from statistical ensembles of measurements which are interpreted as fuzzy signals. The use of fuzzy inputs is of high advantage if the distribution of the signals measured provides essential information about the process to be controlled. In order to solve the problem how a fuzzy signal could be directly processed by the controller some important operations regarding instationary fuzzy signals, especially the differentiation of a fuzzy set with respect to time, have been defined.

A last topic dealt with fuzzy inputs within a Takagi/Sugeno controller. In this connection the antecedence part is provided by the fuzzy input signal whereas the consequence part works with the defuzzified fuzzy signal. Independent of the rule weights and the properties of the fuzzy signals the system of discourse was checked on local and strong global stability.

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Dr. Rainer Palm
Siemens AG R&D, Dept. ZFE ST SN 4
Otto-Hahn-Ring 6, D-81739 Munich, Germany
Tel: +49 89 636 43764
Fax: +49 89 636 3320
email: palm@zfe.siemens.de