

THE TRIAD OF FUZZY THEORY

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Let us imagine that we had a given paradigm of 'lighter' and 'darker' in the shape of a white and black patch, and now, so to speak, we use it to deduce that red is darker than white.
-- Ludwig Wittgenstein [7]

Abstract

Much has been written concerning fuzzy theory in recent years. To the uninitiated, and to many of the initiated as well, the picture which emerges may appear unclear and often contradictory. This is frustrating, but not surprising. The following attempts to clarify some of the more befuddling aspects of fuzzy theory by presenting a representation of fuzzy theory and its relation to classical theory, illuminating semantic interpretations of fuzzy theory, discussing the assumption of burden of proof in fuzzy theory arguments by providing a trial law analogy, and concluding with a view toward the broad implications of fuzzy theory.

Introduction

It is somewhat ironic that the strongest supporters of fuzzy theory hail from a field known for its discrete, binary pursuits. Computer science, with few exceptions, embraces the determinism provided by discrete binary methods problem-solving. Much of the confusion which surrounds fuzzy theory may be attributed to its espousal within such an incongruous environment.

Deterministic approaches in computer science are dictated in large part by hardware. Indeed, it is doubtful whether computers could have reached their current ubiquitousness without the reliability provided by binary machines. The specialization of analog machines demands an inordinate amount of both economic and personnel investment. It is, therefore, not surprising to find that the field has relied on those deterministic and bimodal theories which made the field viable, to the near exclusion of all other approaches to problem-solving. This reliance is showing some signs of waning, however, as other paradigms infiltrate and show themselves useful to the field¹. Fuzzy theory is such a paradigm.

The Fuzzy Triad

Confusion in the understanding of things 'fuzzy' lies not merely in the representation and applications of fuzzy techniques but also in their theoretical relationships. Fuzzy theory might best be depicted as a triad of fuzzy branches, roughly modelled upon the classical triad presented by Russell & Whitehead in *Principia Mathematica* [8].

The fuzzy triad consists of fuzzy logic, fuzzy set theory and fuzzy mathematics [1]-[5], [9]. Fuzzy logic refers to a set of logical tools which include conjunction, disjunction, negation, and implication. Fuzzy set theory provides a representational and relational scheme for the items that fuzzy logic may operate upon. The fuzzy logic is often disguised in fuzzy set theory by the relational operators intersection, union, complement, and subset. Fuzzy mathematics refers to specific techniques, theorems, or methods which are generated by the application of fuzzy logic to fuzzy sets. This framework seems a proper expression of fuzzy theory, the meta-theory which links the triad. The two theories, sharing the same conceptual skeleton, can therefore be represented in a similar fashion.

Perhaps the least understood factor concerning fuzzy theory is that fuzzy set theory is a generalization of classical set theory. It is unfortunate, and somewhat defeatist to have to make such a clarification, but until fuzzy theory is widely accepted as a generalization, such clarification remains necessary.

Simply put, classical sets are fuzzy sets in that they are based upon set membership functions that map onto the range of 0 to 1 inclusively. Classical sets limit the range of set membership to 0 and 1, a subrange of fuzzy set membership functions, leaving classical set theory a specialization of fuzzy theory. It is therefore something of a red herring to present classical set theory and fuzzy set theory as divergent. More properly, once classical set theory is understood to be a specialization of fuzzy set theory, it follows that the results of classical set theory are specializations of the results of fuzzy set theory.

Acceptance of the preceding representations of fuzzy theory and classical theory and of the observation that fuzzy theory is a generalization of classical theory, still leaves the question, "How is the fuzzy theory transformed into classical theory?"

The bulk of the work in unifying fuzzy set theory remains in generalizing the results of classical set theory, namely the identification and definition of general theorems that correspond to the well-accepted theorems of classical set theory which pervade all fields of modern science.

There are, at minimum, two crucial limitations, upon which all aspects of the specialization are dependent:

Fuzzy Logic => Classical Logic

Fuzzy logic does not rely upon the acceptance of the principle of the excluded middle, while classical logic is dependent upon this principle.

Fuzzy Set Theory => Classical Set Theory

The range of the set membership function in fuzzy set theory is limited from 0 to 1 inclusive, while in classical set theory the range of set membership is restricted to either 0 or 1.

These alterations are closely related. It can be argued that the principle of the excluded middle has been dropped from the fuzzy logic, consequently dictating a scheme of variant set membership. Perhaps as convincingly, an alternative argument might be proposed that since the range of set membership functions has been expanded, and truth-values can be mapped into fuzzy sets, the principle of bivalence must be dropped.

In a very real sense, which alteration preceded and which followed is of little but historical interest. Both are found at the same level of abstraction in the theory, and therefore may be viewed in the practical sense as concurrent. Both are needed, as the above arguments suggest, and are found-- sometimes explicitly, but more often implicitly--in current fuzzy theory.

The alterations mentioned above are by no means trivial. The mandates of fuzzy theory have rocked classical mathematics and the numerous fields which rely upon a foundation of classical mathematics. Additionally, since theorems are derived from axioms and fundamental principles, many of the theorems and corollaries which have provided support and a means of proof for a wide range of researchers must be derived again as a result of these fundamental modifications. An abbreviated list of the application fields which require nearly a complete reworking in order to use fuzzy theory might include: linguistics; propositional logic, predicate logic, and all bimodal truth logics; item property representations; probability, and all areas reliant upon statistical analysis; decidability and program verification techniques. In short, nearly all fields which rely on mathematics as a conceptual and theoretical basis are potential targets.

If it is determined that the acceptance of fuzzy theory within a particular application field is beneficial, then much effort is likely to be required in order to provide theoretical and applicational consistency. I should not be construed, in any way, that *all* paradigms which rest upon classical mathematics *must* be redefined. It is becoming increasingly evident, however, that many areas may greatly benefit from the acceptance of fuzzy theory, justifying the work required to codify this acceptance.

Interpretations of Fuzzy Theory

As mentioned above, much of the confusion related to fuzzy theory rests on its generalizations of previously sacrosanct presumptions. Another source of confusion must be attributed to the many and various representations of fuzzy theory that have been thus far provided.

The diversity of fuzzy theory representations is, in large part, a reflection of the far reaching implications that fuzzy theory imposes. Since the attempts by Russell and Whitehead in *Principia Mathematica* [8] to show logic and set theory as the foundations for all of mathematics, much of the work done in mathematics has rested upon set theory. Though some difficulties remain, e.g. Russell's Paradox, bivalent set membership has been accepted as the kernel for nearly all areas of mathematical study. The difficulty and reluctance of accepting fuzzy theory demonstrates that, in large measure, Russell and Whitehead were successful in their monumental task².

To the degree Russell and Whitehead's propositions are accepted by a particular discipline, nearly the same degree of resistance can be anticipated for fuzzy theory. This apparent direct relationship between the degree of resistance to fuzzy theory and the degree of crisp set theory dependence, appears to pervade all fields of study which rest upon crisp mathematics as a foundation. This is, in short, nearly all branches of science. Some branches, e.g. quantum mechanics, which find classification as a central research pursuit, might be impacted in a more direct fashion.

Realizing the omnipresence of modern mathematics, and its reliance on the basic axiom that set membership is bimodal, the reluctance to accept the alterations dictated by fuzzy theory should be expected. The path toward well reasoned consideration has also not been smoothed by the snarl of confusion concerning fuzzy representation and semantic descriptions of the theory.

The breadth and depth of the impact of fuzzy alterations is directly responsible for the wide variety of semantic presentations of fuzzy theory. During the emergence of new scientific and mathematical theories, attempts at providing applications for a theory often appear prior to the full understanding of the theory. Such is the case in fuzzy theory representation: much work has been conducted in providing models for use, and relatively little in providing a concise, unified representation for the theory. The debate concerning "proper" approaches to scientific development continues to rage. The process of introducing applications prior to the full development of a theory will be neither praised nor admonished here.

As most current scientific models depend on crisp set membership functions, it is a matter of practicality that fuzzy theory must show itself of value in applications. In those instances where fuzzy approaches are seen as tools for solving application problems which have been successfully dealt with by crisp paradigms, the burden of proof of the superior handling of the problem with a fuzzy approach lies with those who propose a change. Thus, many of the discussions in fuzzy theory have concerned themselves with attempts to show fuzzy to be more "successful" and "better" than accepted approaches.

In the attempt to show the improvements to be realized from using fuzzy techniques, a multitude of representations of the theory have been presented with little or no correlation vice the fuzzy set membership function. Though in a practical sense, various representations must be provided to show the theory useful, such a wide variety of semantic explanations does not further the effort toward a coordinated representation of the theory.

Given the number and diversity of uses and representations of classical set theory, it should come as no surprise that fuzzy set theory has at least as many representations and/or uses and that many appear divergent. The confusion created by fuzzy semantic representations finds its source in the inductive approach of trying to understand fuzzy theory through the semantics rather than attempting to understand the semantic representations through the theory. Semantic representations for fuzzy applications are inherently dependent upon the application under discussion. Thus there are at least as many semantic representations of fuzzy set applications as there are for crisp set applications. Furthermore, none of them may be judged right or wrong, except as they relate to the field of application. Simply put, fuzzy theory requires no semantic representation; however, as in any theory, semantic labelling is required to demonstrate that the theory actually provides support for the application method.

The many and seemingly disparate characterizations of fuzzy sets are both understandable and unavoidable. New semantic representations should not be discouraged, but fuzzy theory cannot be characterized by a single semantic interpretation, and any attempts to define fuzzy theory in such a manner should be avoided.

The Burden of Proof

The assumption of the burden of proof seems spontaneous in most discussions involving fuzzy methods. In some instances this is both proper and desirable, in others, however, it is neither.

If we are to believe, as Descartes [6], that doubt initiates all reasoning, then certainly doubt initiates all arguments. Argumentation might best be evinced as the process of removing doubt in a second party. Though fuzzy methods are quite adept at handling doubt, arguments in the meta-theory that support fuzzy methods are often found lacking. At least some degree of this failure is due to the unfair assumption of the burden of proof on the part of fuzzy supporters.

In those situations where evidence is being proffered for supplanting extant and acceptable methods, supporters of a new, fuzzy-based solution should rightly bear the burden of proof. Another category of circumstances involves situations in which suitable and accepted solutions do not exist. In these cases the criterium shifts from "better than" to "acceptable". the burden of proof might also be expected to shift toward an equal distribution.

There is yet another class of arguments which attempt to apply fuzzy techniques to broad categories of study. A better understanding of these cases might benefit from noting an analog: the development of scientific techniques in the U.S. judicial system. In the judicial system, much like current trends or scientific evolution, we find a rich body of research material including the notions of proof and precedence.

Though expansive laws might be promulgated by legislation, the interpretation and application of particular laws to particular cases

must be determined by judge and jury. If scientific development in general and the acceptance of fuzzy theory specifically is viewed as a series of trials with fuzzy theory as the

defendant/plaintiff, authors as prosecution/defense attorneys, and the scientific community as judge/jury, then an accurate picture of proper argumentation unfolds.

Most legal cases, like most considerations of the application of fuzzy techniques, do not move beyond the confines of the particular case. On infrequent occasions a point is raised which has implications for a broad range of applications. Such "cases" may result in an appeal to a higher court. Those higher court appeals, while ostensibly focusing on the case at hand, always maintain an awareness of broader implications. Because of their potential impact, such cases are usually not considered. Analogously, attempts will sometimes be made to argue for the broad applicability of fuzzy techniques. As is the case for legal appeals, such arguments must be undertaken only when a true need arises. Because of their potentially broad implications, decisions in such circumstances are attained only laboriously and under stringent requirements of evidence and careful consideration.

The success seen for fuzzy techniques thus far, may depend upon the particular selected domains of application. It is important to show that fuzzy theory provides a consistent and successful approach to solving particular problems. A nice example of a specific argument supporting the use of fuzzy can be found in Cios, Shin, and Goodenday's work concerning the diagnosis of coronary maladies. It is also of long term importance to demonstrate a breadth and consistency in the applications of fuzzy theory. It is important to recognize that arguments which attempt such demonstrations in broad sweeps should be undertaken sparingly and evaluated with due caution and deliberation.

Broad Implications

Only a portion of broad and multifarious implications of fuzzy theory have as yet been revealed. The breadth of its potential impact may at least partially explain the apparent resistance to the fuzzy approach. In addition, some arguments have been made that attempt to show a cultural and sociological link with the acceptance of fuzzy theory. Certainly, few fields have warmly accepted fuzzy theory.

Perhaps the most profound implications have been, and will continue to be, uncovered in the areas of formal logic, and human reasoning models. These areas are not disparate, as formal logic has enjoyed the position of the most widely accepted model for human reasoning, though little evidence has been provided that this is indeed the case.

It is not surprising that a theory which encompasses all the work thus far accomplished in classical theory, and promises to accomplish goals shown to be unattainable by classical approaches, should have such a diverse domain of application.

Endnotes

1. Constructive and intuitive approaches to mathematics deny the logical principle of the excluded middle, cf. Dummett, M. Elements of Intuitionism, Oxford, U, Press, 1977.
2. Neither Russell nor Whitehead are historically considered to be set theorists. Their presentation of the theory of types in Principia

Mathematica was found lacking, but the underlying idea of deducing mathematics from set theory and logic remains their greatest contribution.

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by Ed Deaton
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