Fuzzy Logic Introduction

by Martin Hellmann, March 2001

1.Introduction

Fuzzy Logic was initiated in 1965 (1), (2), (3), by Lotfi A. Zadeh , professor for computer science at the University of California in Berkeley.

Basically, Fuzzy Logic (FL) is a multivalued logic, that allows intermediate values to be defined between conventional evaluations like true/false, yes/no, high/low, etc. Notions like rather tall or very fast can be formulated mathematically and processed by computers, in order to apply a more human-like way of thinking in the programming of computers(4).

Fuzzy systems is an alternative to traditional notions of set membership and logic that has its origins in ancient Greek philosophy. The precision of mathematics owes its success in large part to the efforts of Aristotle and the philosophers who preceded him. In their efforts to devise a concise theory of logic, and later mathematics, the so-called "Laws of Thought" were posited (5). One of these, the "Law of the Excluded Middle," states that every proposition must either be True or False. Even when Parminedes proposed the first version of this law (around 400 B.C.) there were strong and immediate objections: for example, Heraclitus proposed that things could be simultaneously True and not True. It was Plato who laid the foundation for what would become fuzzy logic, indicating that there was a third region (beyond True and False) where these opposites "tumbled about." Other, more modern philosophers echoed his sentiments, notably Hegel, Marx, and Engels. But it was Lukasiewicz who first proposed a systematic alternative to the bi-valued logic of Aristotle (6). Even in the present time some Greeks are still outstanding examples for fussiness and fuzziness, (note the connection to logic got lost somewhere during the last 2 mileniums (7)).

Fuzzy Logic has emerged as a a profitable tool for the controlling and steering of of systems and complex industrial processes, as well as for household and entertainment electronics, as well as for other expert systems and applications like the classification of SAR data.

2. Fuzzy Sets and Crisp Sets

The very basic notion of fuzzy systems is a fuzzy (sub)set. In classical mathematics we are familiar with what we call *crisp sets*. For example, the possible interferometric coherence values are the set *X* of all real numbers between 0 and 1. From this set *X* a subset *A* can be defined, (e.g. all values $0 < \gamma < 0.2$). The *characteristic function* of A, (i.e. this function assigns a number

1 or 0 to each element in *X,* depending on whether the element is in the subset *A* or not) is shown in Fig.1.

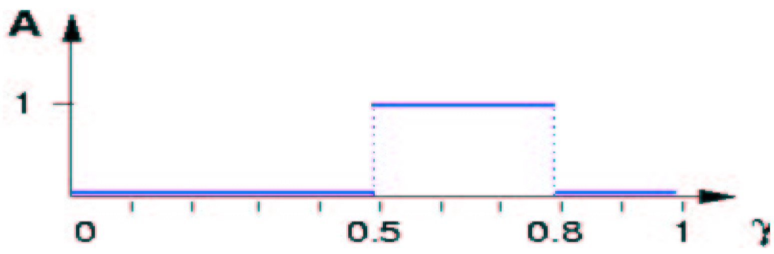

Figure 1. Characteristic Function of a Crisp Set

The elements which have been assigned the number 1 can be interpreted as the elements that are in the set *A* and the elements which have assigned the number 0 as the elements that are not in the set *A*. This concept is sufficient for many areas of applications, but it can easily be seen, that it lacks in flexibility for some applications like classification of remotely sensed data analysis.

For example it is well known that water shows low interferometric coherence γ in SAR images. Since γ starts at 0, the lower range of this set ought to be clear. The upper range, on the other hand, is rather hard to define. As a first attempt, we set the upper range to 0.2. Therefore we get B as a crisp interval B=(0,0.2). But this means that a γ value of 0.20 is low but a γ value of 0.21 not. Obviously, this is a structural problem, for if we moved the upper boundary of the range from γ =0.20 to an arbitrary point we can pose the same question. A more natural way to construct the set *B* would be to relax the strict separation between *low* and *not low*. This can be done by allowing not only the (*crisp*) decision *Yes/No*, but more flexible rules like " *fairly low*". A fuzzy set allows us to define such a notion.

The aim is to use fuzzy sets in order to make computers more intelligent, therefore, the idea above has to be coded more formally. In the example, all the elements were coded with 0 or 1. A straight way to generalize this concept, is to allow more values between 0 and 1. In fact infinitely many alternatives can be allowed between 0 and 1, namely the unit interval I = (0, 1).

The interpretation of the numbers, now assigned to all elements is much more difficult. Of course, again the number 1 assigned to an element means, that the element is in the set *B* and 0 means that the element is definitely not in the set *B*. All other values mean a gradual membership to the set *B.* This is shown in Fig. 2. The *membership function* is a graphical representation of the magnitude of participation of each input. It associates a weighting with each of the inputs that are processed, define functional overlap between inputs, and ultimately determines an output response. The rules use the input membership values as weighting factors to determine their influence on the fuzzy output sets of the final output conclusion.

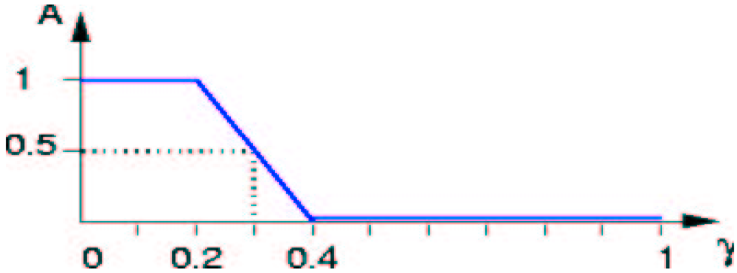

Figure 2. Characteristic Function of a Fuzzy Set

The membership function, operating in this case on the fuzzy set of interferometric coherence γ, returns a value between 0.0 and 1.0. For example, an interferometric coherence γ of 0.3 has a membership of 0.5 to the set *low coherence* (see Fig. 2).

It is important to point out the distinction between fuzzy logic and probability. Both operate over the same numeric range, and have similar values: 0.0 representing *False* (or non–membership), and 1.0 representing *True* (or full–membership). However, there is a distinction to be made between the two statements: The probabilistic approach yields the natural–language statement, "There is an 50% chance that γ is low," while the fuzzy terminology corresponds to "γ's degree of membership within the set of low interferometric coherence is 0.50." The semantic difference is significant: the first view supposes that γ is or is not low; it is just that we only have an 50% chance of knowing which set it is in. By contrast, fuzzy terminology supposes that γ is "more or less" low, or in some other term corresponding to the value of 0.50.

4. Operations on Fuzzy Sets

We can introduce basic operations on fuzzy sets. Similar to the operations on crisp sets we also want to *intersect*, *unify* and *negate* fuzzy sets. In his very first paper about fuzzy sets (1), L. A. Zadeh suggested the *minimum operator* for the intersection and the *maximum operator* for the union of two fuzzy sets. It can be shown that these operators coincide with the crisp unification, and intersection if we only consider the membership degrees 0 and 1. For example, if *A* is a fuzzy interval between 5 and 8 and *B* be a fuzzy number about 4 as shown in the Figure below

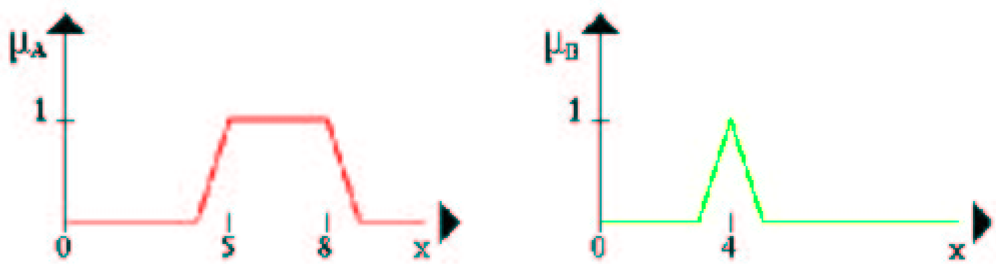

Figure 3. Example fuzzy sets

In this case, the fuzzy set between 5 and 8 ***AND*** about 4 is

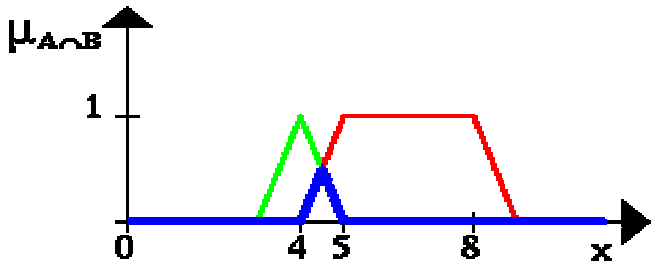

Figure 4. Fuzzy ***AND***

set between 5 and 8 ***OR*** about 4 is shown in the next figure

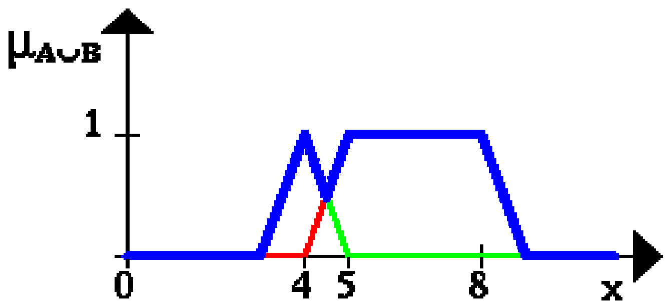

Figure 5. Fuzzy ***OR***

the NEGATION of the fuzzy set A is shown below

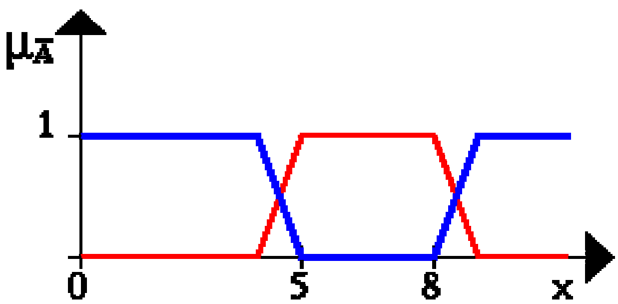

Figure 5. Fuzzy ***NEGATION***

5. Fuzzy Classification

Fuzzy classifiers are one application of fuzzy theory. *Expert knowledge* is used and can be expressed in a very natural way using linguistic variables , which are described by fuzzy sets . E.g., the polarimetric variables Entropy *H* and α–

angle can be modelled as

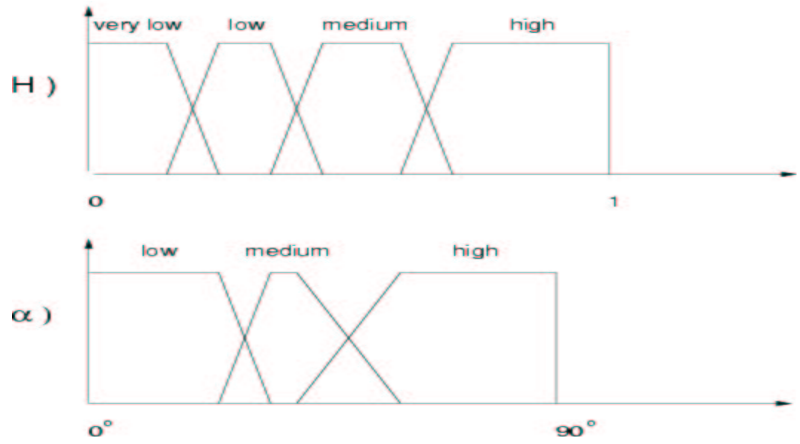

Figure 6. linguistic variables

Now the *expert knowledge* for this variables can be formulated as a rules like

IF Entropy *high* **AND** α *high* **THEN** Class = class 4

The rules can be combinded in a table calles rule base

Entropy	α	***Class***
very low	low	class 1
low	medium	class 2
medium	high	class 3
high	high	class 4

Table 1. Example for a fuzzy rule base

Linguistic rules describing the control system consist of two parts; an antecedent block (between the IF and THEN) and a consequent block (following THEN). Depending on the system, it may not be necessary to evaluate every possible input combination, since some may rarely or never occur. By making this type of evaluation, usually done by an experienced operator, fewer rules can be evaluated, thus simplifying the processing logic and perhaps even improving the fuzzy logic system performance.

The inputs are combined logically using the **AND** operator to produce output response values for all expected inputs. The active conclusions are then combined into a logical sum for each membership function. A firing strength for each output membership function is computed. All that remains is to combine these logical sums in a defuzzification process to produce the crisp output. E.g for a for the rule consequents for each class a so–called *singleton* or a min_max interference can be derived which is the characteristic function of the respective set . E.g. For the input pair of H=0.35 and α=30° the scheme below would apply.

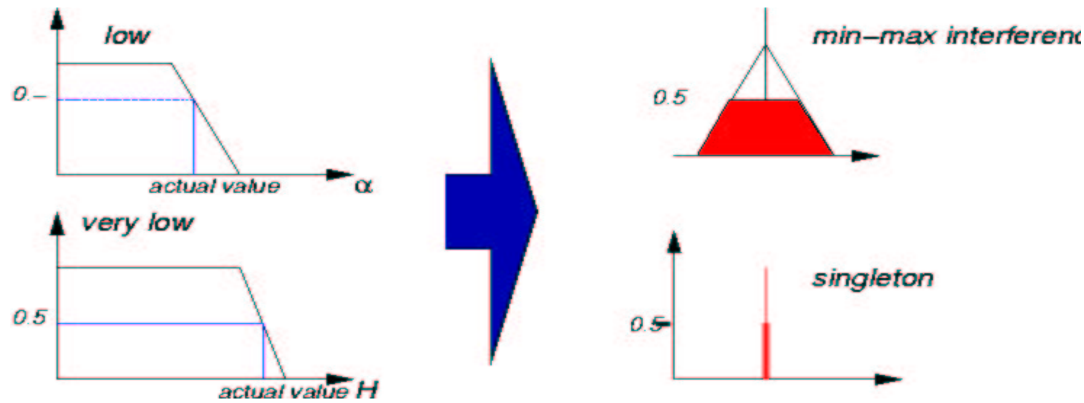

Figure 7. Interference for rule **IF** H *very low* **AND** α *low* **THEN** Class = class 1

The fuzzy outputs for all rules are finally *aggregated* to one fuzzy set.To obtain a crisp decision from this fuzzy output, we have to *defuzzify* the fuzzy set, or the set of singletons. Therefore, we have to choose one representative value as the final output. There are several heuristic methods (*defuzzification methods*), one of them is e.g. to take the center of gravity of the fuzzy set as shown in figure 7., which is widely used for fuzzy sets. For the discrete case with *singletons* usually the the maximum–methodis used where the point with the maximum singleton is chosen.

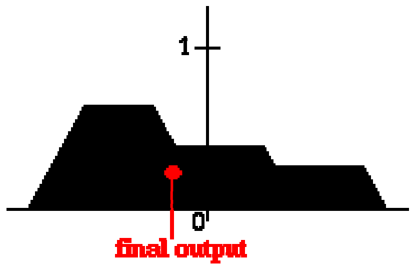

Figure 7. Defuzzyfication using the center of gravity approach

5. Conclusions

Fuzzy Logic provides a different way to approach a control or classification problem. This method focuses on what the system should do rather than trying to modell how it works. One can concentrate on solving the problem rather than trying to model the system mathematically, if that is even possible. On the other hand the fuzzy approach requires a sufficient expert knowledge for the formulation of the rule base, the combination of the sets and the defuzzification. In General, the employment of fuzzy logic might be helpful, for very complex processes, when there is no simple mathematical model (e.g. Inversion problems), for highly nonlinear processes or if the processing of (linguistically formulated) expert knowledge is to be performed. According to literature the employment of fuzzy logic is not recommendable, if the conventional approach yields a satisfying result, an easily solvable and adequate mathematical model already exists , or the problem is not solvable ;)

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