A Fuzzy Model-Based Optimal Control Strategy

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ABSTRACT

In the present contribution we develop a single input – single output fuzzy model–based method for the design and synthesis of optimal tracking control systems. Optimal tracking control systems represent a particular class of quadratic control that are based on a pure quadratic tracking error. criterion. The resulting control strategy maintains the output close to a pre-specified set-point. The control strategy is continuously adapted by a receding horizon predictive algorithm that computes at each time step an adjustment for the extended control sequence and applies effectively only the first value of this sequence. The key idea of the algorithm, is the derivation of the adjoint system associated with a fuzzy dynamic system. This adjoint system can be represented as a fuzzy system so that, only linguistic models are used. The main advantage of this approach is the extension of optimal control results in order to handle fuzzy dynamic systems that can be the only available description of a possibly highly nonlinear plant. An adaptive version of this algorithm can be easily developed and implemented. Numerical simulation results illustrate the applicability of the approach to a simple indoor climate control system.

Keywords: Fuzzy logic, Optimal control, Tracking problem, Fuzzy adjoint system, Receding horizon optimization, Temperature control application.

Introduction

The specification of a control problem can proceed by identifying the control objective with the optimization of a performance index. Usually the performance index is an integral criterion that is evaluated over a predictive horizon of time. The optimization can be handled either as a mathematical programming problem [11] or using results that make use of the special structure of a predictive control optimization problem that expresses the solution in a recursive form. The straightforward approach to predictive control as a mathematical programming problem is limited by the required memory that is of the order of $O(n^2)$ in the case of second order minimization algorithms and of the order $O(n)$ in the case of first order algorithms. In either case the memory requirements are increasing rapidly with the dimension of the prediction horizon. A solution obtained by using the special recursive structure of a predictive control problem can be issued either by the dynamic programming method or by minimum principle of Pontriaghin [12]. The dynamic programming method has the advantage that is conceptually simple and does not need an analytic description of the dynamic system, so that a linguistic (or fuzzy) description can be used instead. This remark was effectively known for already some time (see [13,14]) but only lately has been detailed in a series of applications [8], due only to increasing available computing power. But this approach suffers from what is known as the "dimensionally course" as the memory requirements are increasing exponentially with the dimension of the problem that is proportional to the length of the prediction horizon. So that, even with all the tricks of the trade that are used to simplify this approach, the computational complexity of involved by the final implementation remains considerable.

A much more interesting approach is issued from the minimum principle that leads to necessary optimum conditions, and to equations that can be solved either analytically or numerically.. However the main difficulty that is envisaged in the application of the minimum principle for a fuzzy system, is the fact that analytical representation of the system is required. The main result presented in this paper allows the formulation of the minimum principle for plants where only a fuzzy description is available and leads to a numerical solution for the control strategy.

For the lack of simplicity we treat here only a special class of predictive control problems where the criterion to be minimized is the quadratic error between a pre-scheduled tracking trajectory and the systems trajectory. These kind of control problems are conventionally named as tracking problems (see [12] and [13]). We focus here on the solution of such problems for the case when the model of the plant is given by a qualitative model expressed in terms of a fuzzy logic rule base and parametrized by correspondingly defined fuzzy sets. The main theoretical points are the derivation of the adjoint system of a fuzzy system and of the gradient correction of the current control strategy. This correction is realized as the back-propagation of the tracking error through the adjoint system integrated in reverse time. The general conditions given by the minimum principle [11] are adapted to representation of fuzzy dynamic systems leading to the derivation of the adjoint of a fuzzy system. What is particularly interesting is that this adjoint system can be expressed as a fuzzy system and it can be obtained by simple manipulations from the initial fuzzy model of the plant.

The idea of extending analytical results of predictive control theory for the fuzzy systems has already been used in [5], where a very simple Predictive PID control is developed. A generalized predictive control-like algorithm is further developed in [9] or in [10], where different gradient corrections of the control strategy are used. But the correction is computed on all the horizon rather than recursively. The idea of computing the adjoint of a fuzzy dynamically system and use it for predictive control was introduced in[15]. In the present paper we give a fuzzy representation for the resulting equations of the adjoint system by introducing the notion of the fuzzy sensitivity membership functions associated with the fuzzy partitions describing the system. The connex results are used to infer the description of the adjoint of a dynamic system, but the algorithm is much more general and can be used to obtain a general fuzzy representation for the differential operators applied to fuzzy functions. We also introduce a fuzzy adaptation mechanism for the gain of the predictive control adjusting strategy. Finally we show how the tracking algorithm can be used in order to achieve predictive disturbance rejection, for the processes where the disturbances are known in advance. Precise knowledge of the future perturbations affecting the system is not necessary, and can be replaced with a qualitative description of these perturbation, given in terms of fuzzy sets.

Fuzzy Function Representation

Fuzzy sets were introduced by Zadeh in 1965 as an appropriate way to represent imprecision and incertitude. In the seminal paper [1] Zadeh introduced the object of a fuzzy set as a set with no definite frontier that is defined by its membership function and characterised the operations between the fuzzy sets in terms of the membership functions. This is the basic idea of the fuzzy theory. Every element of a fuzzy set is thus characterised by a degree of membership to the fuzzy set that can take any value in the interval [0, 1] as compared with an usual set where an element is either in the set or out of it. A usual, "crisp" set is thus a limit case of a fuzzy set, and thus a fuzzy set is a natural extension of crisp sets. A further development of these ideas was done by Zadeh in 1968 [2] and 1973 where there were laid the basis of the description of complex systems in terms of the new fuzzy language. This new tool is particularly adapted for modeling the evolution of complex and imprecise systems. A precise numerical parametrization of the linguistic orders of magnitude is defined by a *fuzzy partition* of the universe of discourse X , that is a collection $\tilde{X}$ of fuzzy sets: $\tilde{X} = \{X_1, X_2, \ldots, X_{nx}\}$ that satisfies the normalization relation: $\sum_{i=1}^{nx} \mu_{X_i}(x) = 1$ for any x. This condition is imposed by the use of the special fuzzy logic connectors that are described further.

A fuzzy representation of a function f describes the input-output behavior of an analytical unknown function, by assigning fuzzy values in the output domain Y to crisp values from the input domain X, by means of the fuzzy inference mechanism. The algebraic interpretation of the multi input fuzzy inference is that starting from fuzzy partitions defined component-wise on each of component sub-domains of the input universe X, one constructs a global fuzzy partition for the all the universe X. This partition is thereafter used to compute the firing power of each rule in the rule base, that associates to any input hypothesis an output consequence that can be either a fuzzy set or a crisp value. The rule base defines the fuzzy description of the input-output behavior of the function f.

A simplified representation of the function f can obtained if we make several hypothesis that we will describe in what will follow.

The fuzzification operation is given by a singleton $\sigma(x^*)$ that is a fuzzy set the is naturally associated with a crisp value and is defined as:

$$\mu_{\sigma(x^*)}(x) = \delta(x - x^*) = \begin{cases} 0 \ if \ x \neq x^* \\ 1 \ if \ x = x^* \end{cases} \qquad (1)$$

The output fuzzy partition $\tilde{Y}$ consists only of singletons: $\tilde{Y} = \{\sigma(y_j)\}_{j=1,ny}$ This is known as Sugeno's method ([3]). and is the preferred in the cases where the simplicity of the implementation is included as a design constraint. We also consider the case where there is only a unique rule that is activated for each hypothesis. This means that the rule strengths are either 0 or 1. The fuzzy operators are chosen as $\mu_{P \wedge Q} = \mu_P \cdot \mu_Q$ and $\mu_{P \vee Q} = \mu_P + \mu_Q$. These forms of the fuzzy connectors are greatly simplifying the calculations, especially for the defuzzification, but they impose that the normalization condition of the fuzzy partition is strictly verified. We obtain, under these hypothesis, a simple description for the input-output behavior for the fuzzy functional representation. The general formula of a fuzzy function is given by:

$$y^* = f(x^*) = \sum_{i=(i^1,\ldots,i^N)} \rho_{(i^1,\ldots,i^N)}(x^{*1},\ldots,x^{*N}) \cdot \varphi(x^1_{i^1},\ldots,x^N_{i^N},x^*) \qquad (2)$$

that represents a *multidimensional interpolation* between the local functional approximations $\varphi(x^1_{i^1},\ldots,x^N_{i^N},x^*)$ of the function f, defined on an N-dimensional grid of points: $x_i = (x^1_{i^1},\ldots,x^N_{i^N})$. It holds an obvious relation:

$$\varphi(x^1_{i^1},\ldots,x^N_{i^N},x^* = x^1_{i^1},\ldots,x^N_{i^N}) = f(x^1_{i^1},\ldots,x^N_{i^N}) \qquad (3)$$

Two choices are usual for the functions $\varphi(x^1_{i^1},\ldots,x^N_{i^N},x^*)$ that can be either linear (as in [17]) or simply linguistically labeled constants that represent the qualitative values of the function f in the points of the grid: $\varphi(x^1_{i^1},\ldots,x^N_{i^N},x^*) = f(x^1_{i^1},\ldots,x^N_{i^N})$.

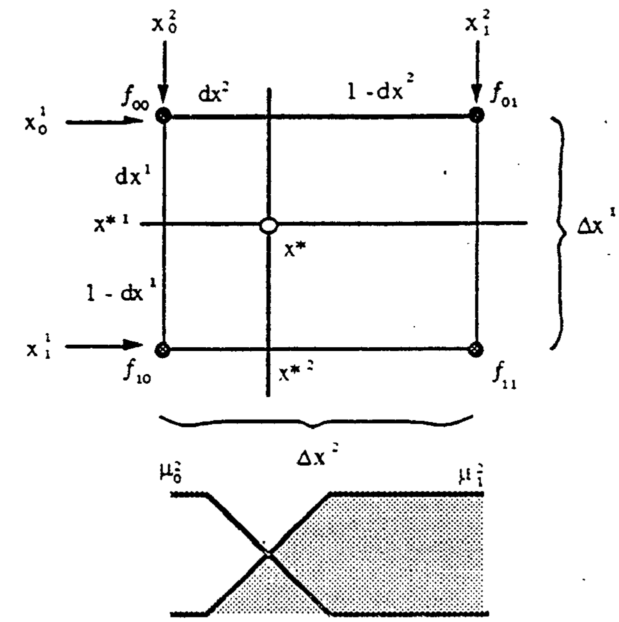

Fig 1 Fuzzy function representation on a fuzzy multidimensional interpolation grid

In order to compute the partial derivatives of the resulting fuzzy function we will use the specific component wise structure of the fuzzy interpolation scheme. The main results are presented in the following:

$$\left.\frac{\partial f(x^1,\ldots,x^N)}{\partial x^k}\right|_{(x^{*1},\ldots,x^{*N})} = \qquad (3)$$

$$\sum_{i=(i^1,\ldots,i^N)} \left.\frac{\partial \rho_{(i^1,\ldots,i^N)}(x^1,\ldots,x^N)}{\partial x^k}\right|_{(x^{*1},\ldots,x^{*N})} \cdot \varphi(x^1_{i^1},\ldots,x^N_{i^N},x^*)$$

$$+ \sum_{i=(i^1,\ldots,i^N)} \rho_{(i^1,\ldots,i^N)}(x^{*1},\ldots,x^{*N}) \cdot \left.\frac{\partial \varphi(x^1_{i^1},\ldots,x^N_{i^N},x^1,\ldots,x^N)}{\partial x^k}\right|_{(x^{*1},\ldots,x^{*N})}$$

With the specific choice of the "and" operator as: $\mu_{P \wedge Q} = \mu_P \cdot \mu_Q$ the interpolation coefficients $\rho_{(i^1,\ldots,i^N)}$ become multilinear in the component wise independent fuzzy membership functions $\mu_{X^p_{i^p}}(x^{*p})$:

$$\rho_{(i^1,\ldots,i^N)}(x^{*1},\ldots,x^{*N}) = \mu_{X^1_{i^1}}(x^{*1}) \cdot \ldots \cdot \mu_{X^N_{i^N}}(x^{*N}) \qquad (4)$$

so that the first term of equation (3) becomes:

$$= \sum_{i=(i^1,\ldots,i^N)} \mu_{X^1_{i^1}}(x^{*1}) \cdots \left.\frac{\partial \mu_{X^i_{i^i}}(x^i)}{\partial x^i}\right|_{(x^{*i})} \cdots \mu_{X^N_{i^N}}(x^{*N}) \cdot \varphi(x^1_{i^1},\ldots,x^N_{i^N},x^*)$$

The precedent formula leads to the introduction of the *fuzzy sensitivity membership functions* associated with a fuzzy partition. We first define a *unimodular* fuzzy membership function $\mu_A(x)$ as a membership function that has a connex kernel. We introduce a *fuzzy univariate membership function* as a fuzzy set $\mu_A(x)$ for which $-\frac{\partial}{\partial x}\mu_A(x) \geq 0,\ x > x_c$ and $\frac{\partial}{\partial x}\mu_A(x) \geq 0,\ x < x_c$ are unimodular. With this definitions, for a given fuzzy partition $\sum_{i=1}^{nx}\mu_{X_i}(x) = 1$ the associated *fuzzy sensitivity membership functions* are defined by the formula:

$$\eta_{X_{i,i+1}}(x) = \frac{\frac{\partial}{\partial x}\mu_{X_{i+1}}(x)}{\max\limits_{x\in[x_i,x_{i+1}]}\left(\frac{\partial}{\partial x}\mu_{X_{i+1}}(x)\right)} \quad (\forall)\, x \in [x_i, x_{i+1}] \qquad (5)$$

An example of a fuzzy partition and of the associated *fuzzy sensitivity membership functions* is given in fig (2).

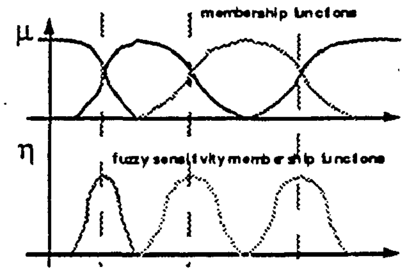

Fig. 2 Fuzzy partition and the associated fuzzy sensitivity membership functions

We remark that the normalization relation: $\mu_{X_i}(x) + \mu_{X_{i+1}}(x) = 1$ implies that the following relation holds true: $\frac{\partial}{\partial x}\mu_{X_i}(x) = -\frac{\partial}{\partial x}\mu_{X_{i+1}}(x)$ (6)

The first term of equation [3] becomes, then:

$$= \sum_{i=(i^1,\ldots,i^N)} \mu_{X_{i^1}^1}(x^{*1})\cdot\ldots\cdot\frac{\partial\mu_{X_{i^i}^i}(x^i)}{\partial x^i}\Bigg|_{(x^{*i})}\cdot\ldots\cdot\mu_{X_{i^N}^N}(x^{*N})\cdot\varphi(x_{i^1}^1,\ldots,x_{i^N}^N,x^*)$$

$$= \mu_{X_{i^1}^1}(x^{*1})\cdot\ldots\cdot\eta_{X_{i^k}^k}(x^{*k})\cdot\ldots\cdot\mu_{X_{i^N}^N}(x^{*N})\cdot\max_{x\in[x_i,x_{i+1}]}\left(\frac{\partial}{\partial x}\mu_{X_{i+1}}(x)\right)\cdot$$
$$\cdot\left[\varphi(x^{*1},\ldots,x_{k+1}^{*k},\ldots,x^{*N},x^*) - \varphi(x^{*1},\ldots,x_k^{*k},\ldots,x^{*N},x^*)\right]$$

$$= \mu_{X_{i^1}^1}(x^{*1})\cdot\ldots\cdot\eta_{X_{i^k}^k}(x^{*k})\cdot\ldots\cdot\mu_{X_{i^N}^N}(x^{*N})\cdot$$
$$\cdot\max_{x\in[x_i,x_{i+1}]}\left(\frac{\partial}{\partial x}\mu_{X_{i+1}}(x)\right)\cdot\left(x_{k+1}^{*k} - x_k^{*k}\right)\cdot$$
$$\cdot\frac{\left[\varphi(x^{*1},\ldots,x_{k+1}^{*k},\ldots,x^{*N},x^*) - \varphi(x^{*1},\ldots,x_k^{*k},\ldots,x^{*N},x^*)\right]}{\left(x_{k+1}^{*k} - x_k^{*k}\right)}$$

If we define the *contrastive factor:*

$$\alpha_{k,k+1} = \max_{x\in[x_i,x_{i+1}]}\left(\frac{\partial}{\partial x}\mu_{X_{i+1}}(x)\right)\cdot\left(x_{k+1}^{*k} - x_k^{*k}\right) \qquad (7)$$

then the preceding formula is rewritten as:

$$= \mu_{X_{i^1}^1}(x^{*1})\cdot\ldots\cdot\eta_{X_{i^k}^k}(x^{*k})\cdot\ldots\cdot\mu_{X_{i^N}^N}(x^{*N})\cdot$$
$$\cdot\alpha_{k,k+1}\cdot\frac{\Delta_k\varphi(x^{*1},\ldots,x_k^{*k},\ldots,x^{*N},x^*)}{\Delta_k x_k^{*k}} \qquad (8)$$

When the fuzzy membership functions are piece wise linear (triangular or trapezoidal), then the fuzzy sensitivity sets are "crisp" sets. The fuzzy derivative of a fuzzy function can be computed by the following steps:

① Compute the *fuzzy sensitivity membership functions* $\eta_{X_{i,i-1}}(x)$ and the *contrastive factors* $\alpha_{k,k+1}$ associated with the fuzzy partition that describe the qualitative scale of the system.

② Compute the continuos partial derivatives of the functional approximations of the function f

$$\frac{\partial\varphi(x_{i^1}^1,\ldots,x_{i^N}^N,x^1,\ldots,x^N)}{\partial x^k}\Bigg|_{(x^{*1},\ldots,x^{*N})}$$

and their discrete partial derivatives

$$\frac{\Delta_k\varphi(x^{*1},\ldots,x_k^{*k},\ldots,x^{*N},x^*)}{\Delta_k x_k^{*k}}$$

③ The final value of the derivative is given by formula [3] and represent the summation of two functions that have the fuzzy function representation.

Fuzzy System Optimality Conditions Given by the Minimum Principle

For the dynamic system described by the following state space equations:

$$x(t+1) = f(x,u,t) \qquad (9)$$

with the initial condition $x(t_0) = x_0$. The state $x \in \Re^n$, the control $u \in \Omega \subset \Re^m$ and f is a function that is sufficiently smooth. We form the cost functional:

$$J = M\left(x(t_f),t_f\right) + \sum_{t=t_0}^{t_f-1} L(x,u,t) \qquad (10)$$

and we formulate the control optimization problem as the minimization of this functional: J = min. We form the associated discrete Hamiltonian function:

$$H(x,u,\lambda,t) = \lambda^T(t+1)\cdot f(x,u,t) + L(x,u,t) \qquad (11)$$

with λ the adjoint (co)vector $\lambda \in \Re^n$. Suppose that the solution to the optimization problem is given by the optimal input $u^*(t)$ that generates the optimal trajectory $x^*(t)$. Then the necessary conditions given by the discrete minimum principle are:

$$\begin{cases} x(t+1) = H_{\lambda(t+1)}(x,u,\lambda,t) \\ \lambda(t) = H_{x(t)}(x,u,\lambda,t) \\ H\left(x^*,u^*,\lambda^*,t\right) = \min\limits_{u\in\Omega} H\left(x^*,u,\lambda^*,t\right) \end{cases} \qquad (12)$$

together with the boundary conditions:

$$0 = [M_x(x(t_f),t_f) - \lambda(t_f)]\cdot\delta x_f +$$
$$+[M_t(x(t_f),t_f) + H(x,u,\lambda,t_f)]\cdot\delta t_f \qquad (13)$$

and for a fixed final time t_f we have the terminal condition: $M_x\left(x(t_f),t_f\right) = \lambda(t_f)$ (14)

Rewriting the first two optimal conditions (8) we obtain:

$$\begin{cases} x(t+1) = f(x(t),u(t),t) \\ \quad x(t_0) = x_0 \\ \lambda(t) = f_x(x,u,t)\cdot\lambda(t+1) + L_x(x(t),u(t),t) \\ \quad \lambda(t_f) = M_x\left(x(t_f),t_f\right) \end{cases} \tag{15}$$

In the case with no constraints, the last of the necessary conditions become:

$$\begin{aligned} 0 &= H_{u(t)}(x,u,\lambda,t) = \\ &= L_u(x,u,t) + f_u(x,u,t)\cdot\lambda(t+1) \end{aligned} \tag{16}$$

We have been using the notation $f_x(x,y) = \left[\dfrac{\partial f(x,y)}{\partial x}\right]^T$.

The Gradient Method for Fuzzy Systems

Albeit some very simple cases of the general optimization problem, that have an analytical explicit solution, one is contrived to use numerical methods to approach the optimal solution. The gradient method is probably the simplest iterative method. If at the iteration step k we have an approximate optimal solution $u^{(k)}(t)$ and the associated trajectory $x^{(k)}(t)$ issued from the integration of the equation $x(t+1) = H_{\lambda(t+1)}(x,u,\lambda,t)$ that respects the initial condition $x(t_0) = x_0$, then the costate vector $\lambda^{(k)}(t)$ that respects the final condition $\lambda(t_f) = M_x\left(x(t_f),t_f\right)$ can be obtain by the integration of equation $\lambda(t) = H_{x(t)}(x,u,\lambda,t)$. The minimum of the functional $H(x,u,\lambda,t)$ can be pursued by searching in the steepest descent direction given by the gradient of the hamiltonian function H :

$$\delta H(x,u,\lambda,t) = \langle H_u(x,u,\lambda,t) | \delta u\rangle \tag{17}$$

so that a reasonable choice may be:

$$\delta u = u^{(k+1)}(t) - u^{(k)}(t) = -\gamma\cdot H_u(x,u,\lambda,t) \tag{18}$$

with $\gamma \geq 0$ a real positive constant that can be found by a monovariable minimum search procedure. The **gradient algorithm** can be implemented with the initialization step $u(t) = u^{(0)}(t)$. Then with $u(t) = u^{(k)}(t)$ integrate the equation: $x(t+1) = f(x,u,t)$ $\quad x(t_0) = x_0$ in order to obtain the trajectory: $x(t) = x^{(k)}(t)$. With $u(t) = u^{(k)}(t)$ and $x(t) = x^{(k)}(t)$ integrate backwards in time the equation: $\lambda(t) = H_{x(t)}(x,u,\lambda,t)$ $\quad , \lambda(t_f) = M_x\left(x(t_f),t_f\right)$

in order to obtain the costate $\lambda(t) = \lambda^{(k)}(t)$. Then we generate the new control:

$$u^{(k+1)}(t) = u^{(k)}(t) - \gamma\cdot H_u(x,u,\lambda,t) \tag{19}$$

The terminating condition is on the norm of the gradient:

$$\|H_u(x,u,\lambda,t)\| \leq \varepsilon_{\min} \tag{20}$$

Fuzzy Model-Based Tracking Systems

Using the canonical fuzzy function representation, a description of a fuzzy dynamic system can be given as a fuzzy version of a non linear discrete system:

$$\begin{aligned} x(t+1) &= f(x(t),u(t),t) \\ y(t) &= h(x(t),t) \end{aligned} \tag{21}$$

with functions f and h defined by their canonical fuzzy function representation. The state of the system is x, the input is u and the output is y.

A *pure tracking problem* can then be defined if in the criterion (6) we take $L(x,u,t)$ as the tracking error between the actual output and the desired output trajectory:

$$L(x,u,t) = \left\| y(t)^* - y(t)\right\|^2 = e(t)^2 \tag{22}$$

Then the equations (11) become:

$$\begin{cases} x(t+1) = f(x(t),u(t),t) \\ \quad x(t_0) = x_0 \\ \lambda(t) = f_x(x,u,t)\cdot\lambda(t+1) - h_x(x(t),u(t),t)\cdot e \\ \quad , \lambda(t_f) = 0 \end{cases} \tag{23}$$

and the gradient correction (14) becomes:

$$\delta u = -\gamma\cdot H_u(x,u,\lambda,t) = -\gamma\cdot f_u(x,u,t)\cdot\lambda(t+1)$$

If the variable x is directly measurable then:

$$\begin{aligned} x(t+1) &= f(x(t),u(t),t) \\ &\quad , x(t_0) = x_0 \end{aligned} \tag{24}$$

$$\begin{aligned} \lambda(t) &= f_x(x,u,t)\cdot\lambda(t+1) - e(t) \\ &\quad , \lambda(t_f) = 0 \end{aligned} \tag{25}$$

The adjoint of a fuzzy dynamic system

The second equation (23) represents the adjoint of the fuzzy system with respect to the tracking criterion. The interpretation of the equation is that of an error λ that is back propagating through a temporal network. The correction in equation is proportional to this error.

In order to compute the derivatives $f_u(x,u,t)$ and $f_x(x,u,t)$ we use the results already obtained. For a time invariant fuzzy system:

$$x(t+1) = f(x(t),u(t)) \tag{26}$$

that is described in terms of rules of the kind:

"`if (x(t) is SMALL) and (u(t) is HIGH)` (27)
`then (x(t+1) is BIG)`"

or

"`if (x(t) is SMALL) and (u(t) is HIGH)` (28)
`then (x(t+1) is` $x(t+1) = \varphi_k(x(t),u(t))$`)`"

or

"`if (x(t) is SMALL) and (u(t) is HIGH)` (29)
`then (x(t+1) is` $x(t+1) = \varphi_k(x(t)) + \gamma_k(x(t))\cdot u(t)$`)`"

at their turn parametrized by the fuzzy partitions:

$\tilde{U} = \{U_1, \dots, U_{nu}\}$ and $\tilde{X} = \{X_1, \dots, X_{nx}\}$

The equation (27) describes a classical fuzzy system, equation (28) a Sugeno fuzzy system and equation (29) a semianalytic system (partially analytic, partially fuzzy).

The gradient algorithm for fuzzy models

First we simulate, for a test control sequence, forward in time, the fuzzy model given by:

$$x(t+1) = f(x(t),u(t),t) \qquad , x(t_0) = x_0$$

in order to obtain the groping trajectory $\{x(t_0),\ldots, x(t_f)\}$. Then we back-propagate the tracking error through the adjoint of the fuzzy system:

$$\lambda(t) = f_x(x,u,t)\cdot\lambda(t+1) - e(t) \qquad , \lambda(t_f) = 0$$

and we obtain the adjoint co-vector λ. The value of $f_x(x,u,t)$ is determined by the eq. (4), identifying the two activated linguistic cells so that $x \in [x_k, x_{k+1}]$

Thereafter we apply the control strategy correction:

$$\begin{aligned}\delta u &= -\gamma \cdot H_u(x,u,\lambda,t)\\ &= -\gamma \cdot f_u(x,u,t)\cdot\lambda(t+1)\end{aligned} \qquad (28)$$

The value of $f_u(x,u,t)$ is determined by the equation (4), identifying the two activated linguistic cells corresponding to the points $u \in [u_k, u_{k+1}]$

Fuzzy adaptation of the gradient gain

The equation (28) include predictive information by the back propagation of the tracking error through the adjoint system over the predictive horizon. However the feedback information is taken into account only through the sliding horizon strategy. We can reinforce the feedback loop by adding a proportional term to the equation (28) that will have an integral effect due to the incremental form of the equation:

$$\begin{aligned}\delta u &= -\gamma \cdot H_u(x,u,\lambda,t)\\ &= -\gamma(e,\Delta e)\cdot f_u(x,u,t)\cdot\lambda(t+1)\end{aligned} \qquad (29)$$

The gradient gain can be adapted by a fuzzy rule set of the form:

"**if (e(t) is SMALL) and (Δe(t) is HIGH)** (30)

then (γ is BIG)"

Predictive disturbance rejection

The process can have a predictable disturbance $\xi(t)$:

$$x(t+1) = f(x(t),\xi(t),u(t),t) \qquad (31)$$

with a possible fuzzy description of the form:

"**if (x(t) is SMALL) and $\xi(t)$ is BIG)** (32)

and (u(t) is HIGH) then (x(t+1) is BIG)"

The advantage is that the order of magnitude of the disturbance $\xi(t)$ must be known in advance only with approximation. Then the gradient control correction is the same (29) but the back propagated error will contain also the information about the time and magnitude of the future perturbation and will adapt the control input accordingly. The disturbance can be predicted as:

"**if (time is SOON) then $\xi(t)$ is BIG)**" (33)

Application Case Study

The algorithm was applied to a building heating process. The temperature inside a building is established as the result of a thermal balance that sums up the heat exchanges with the walls, solar radiative gains through the walls and windows, the renewal of the interior air with exterior fresh air and the heat extracted or dissipated in the room by the cooling or heating devices or by other sources of heat. The actual process is infinite dimensional and is described by a system of partial differential equations coupled by their frontier conditions. However, simplified physical reasoning can provide low order lumped models that are accurate enough for control purposes (see [20]). The process output is the indoor temperature Ti that characterizes the living zone, and the process input is the actual heating or cooling power P that is delivered into the zone. The external temperature Te is a measurable perturbation and the solar radiation or other unknown heating sources gathered in W are non measurable perturbations. A first order nonlinear model of the plant is:

$$\delta T_i = f((T_e - T_i), W, P)$$

that can be described by rules of the form:

if $((T_e - T_i) =$ "*Negative Big*"$) \wedge (P =$ "*Positive Big*"$)$

$\wedge (W =$ "*Zero*"$)$ then $(\delta T_i =$ "*Positive Small*"$)$

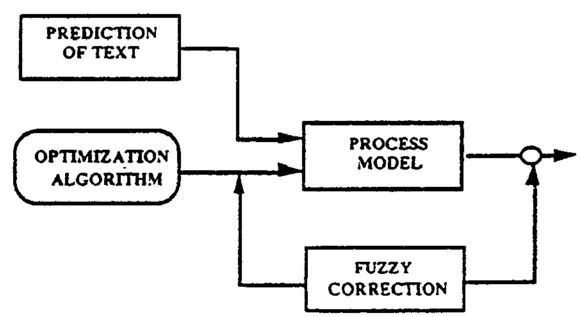

Fig. 3 The Fuzzy model-based predictive control scheme

The control is based on a fuzzy model that is adapted on line in order to compensate the initial modeling errors with which the fuzzy model was acti8vated. This adaptation is done with the fuzzy model adaptation tools described in [6]-[10]. Constain handling strategies can be added to the system in order to deal wit constraints on input and output variables. In Figures (4...6) we show the numerical simmulations of the algorithm.

Conclusion

In the present article a tracking algorithm that is based on fuzzy qualitative models is described The algorithm uses a special fuzzy parametrization of the process model in order to make use of general results from the optimal control theory. The fuzzy model is treated as a nonlinear system for which an adjoint system is build and a receding horizon correction of the control strategy is applied. At each step fuzzy model is predicted over the prediction horizon and the tracking error is back propagated through the adjoint of the fuzzy model in order to determine the correction to the control strategy. The interest of this approach is of showing how advanced predictive control strategies can be applied to fuzzy models.

With respect to other existing predictive control approaches based on the use of fuzzy models for the description of the process, the present approach distinguishes itself by the fact that the numerical complexity that is involved by the control strategy search procedure is considerably alleviated.

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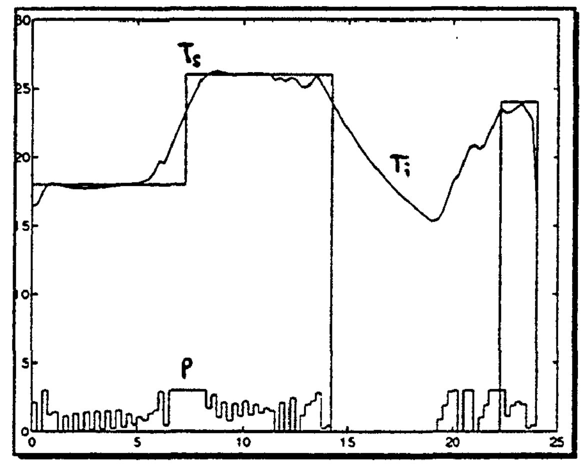

Fig. 4 Predictive control Trajectoryes with constraint handling

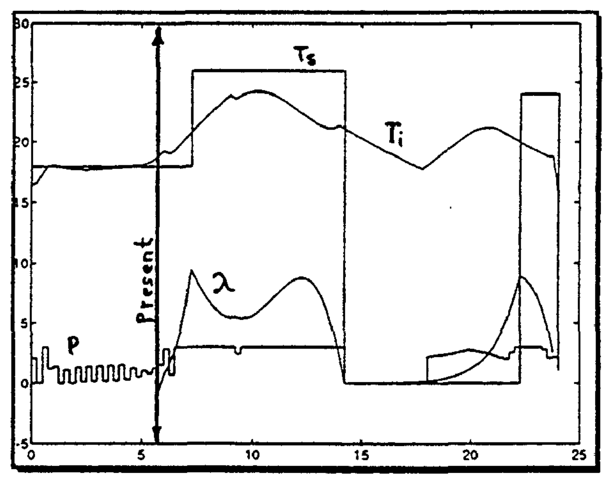

Fig. 5 The adjoint system trajectory used for strategy control correction

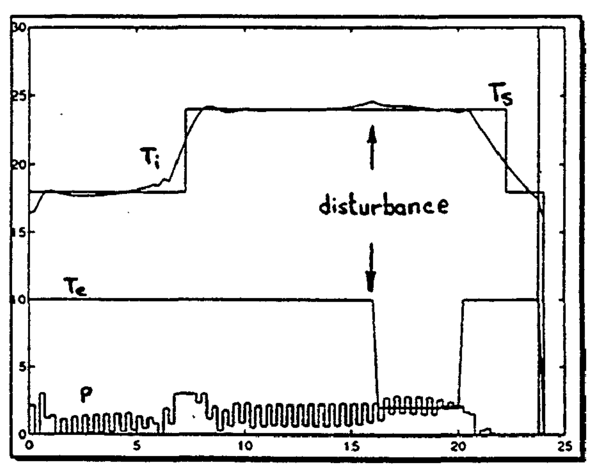

Fig. 6 Predictive disturbance rejection