Performance Analysis of Fuzzy Proportional-Derivative Control Systems

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Abstract

This paper analyzes the performance of a fuzzy proportional-derivative (PD) controller in comparison with the conventional PD controller. The design of the fuzzy PD controller follows the structure of the conventional digital PD controller, with additional fuzzy logic control rules. The resulting controller, therefore, has the same linear structure as that of the conventional digital PD controller, except that both the proportional and the derivative parts have non-constant gains. The fuzzy proportional and derivative gains are nonlinear functions of the control-input signals and hence have a self-tuning control capability. Thus, the proposed fuzzy PD controller preserves the simple linear structure of the conventional PD controller yet enhances its adaptive control capability. In computer simulations, a set of linear systems, with or without time-delays, were used to test the performance of the fuzzy PD controller in [4], and a set of nonlinear systems are used to test the performance of the fuzzy PD controller in this paper. The performance has been compared to the conventional PD controller for the same linear and nonlinear systems. Computer simulation results have demonstrated the advantages of the fuzzy PD controller, particularly if the system to be controlled is nonlinear.

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1. Introduction

One of the main advantages of fuzzy controllers is that they can be used to replace conventional controllers when the system under control is highly complex and/or only linguistically described. With its "rule-based" characteristics, many complex and ill-conditioned control requirements can be implemented in a relatively efficient and inexpensive way.

This paper is an extension of our earlier work on the design of the fuzzy PD controller [4] and its stability analysis. On the other hand, this paper is in parallel to the studies of the fuzzy PI controller design and its stability analysis [1,2]. One of the significant differences of this design from other fuzzy controllers is that the fuzzy logic is used in this design to self-tune (or adjust) the parameters of the controller. This design, however, follows the standard procedure of fuzzy logic controllers design, which consists of (1) fuzzification, (2) fuzzy rule-base establishment, and (3) defuzzification.

The design of the fuzzy PD controller will be briefly described in the next section, while in Section 3 the computer simulation results will be demonstrated.

2. Design of the Fuzzy PD Controller

The fuzzy PD controller was designed by the present authors in [4] used the following notation: Let T, $sp(nT)$, and $y(nT)$ denote the sampling period, the set-point, and the output of the system respectively. The error and the rate of change of the error signals are defined by $e(nT) := y(nT) -$

$sp(nT)$ and $r(nT) := (e(nT) - e(nT-T))/T$, respectively. The fuzzy controller consists of two inputs: $e(nT)$ and $r(nT)$. The output of the fuzzy controller $u(nT)$, which is also the input to the process under control, is given by

$$u(nT) = -u(nT-T) + K_u \Delta u(nT), \quad (1)$$

where K_u is a constant control gain and $\Delta u(nT)$ is an incremental control output, both need to be determined in the design. The block diagram of the closed-loop fuzzy PD control system is illustrated in Fig. 1, where $\mathcal{N}(\cdot)$ is the process under control which can be either linear or nonlinear. For more details, see [4].

The membership functions for the error, the rate of change of the error, and the output of the controller are shown in Fig. 2. The constant $L > 0$ used in the definition of the membership functions is determined by the designer according to the value ranges of the error, rate, and output, and will be fixed after it is chosen.

In [4], it was shown that the membership functions of the error and the rate signals decompose their value ranges into twenty adjacent input-combination (IC) regions as shown in Fig. 3. It was also shown in [4] that when the locations of the error $e(nT)$ and the rate $r(nT)$ are in the regions $IC1, IC2, IC5, IC6$, we have, by the standard "center of mass" defuzzification formula,

$$\Delta u(nT) = \frac{L}{2(2L - K_p|d(nT)|)} [K_p d(nT) - K_d r(nT)], \quad (2)$$

where $d(nT) = [e(nT) + e(nT-T)]/T$. Similarly, if $e(nT)$ and $r(nT)$ are located in the regions $IC3, IC4, IC7, IC8$, then we have

$$\Delta u(nT) = \frac{L}{2(2L - K_d|r(nT)|)} [K_p d(nT) - K_d r(nT)], \quad (3)$$

and in the regions $IC9 - IC20$, we have

$$\begin{aligned}
&\Delta u(nT) \\
&= \frac{1}{2}[L - K_d r(nT)] \quad \text{in IC9, IC10}, \quad (4) \\
&= \frac{1}{2}[-L + K_p d(nT)] \quad \text{in IC11, IC12}, \quad (5) \\
&= \frac{1}{2}[-L - K_d r(nT)] \quad \text{in IC13, IC14}, \quad (6) \\
&= \frac{1}{2}[L + K_p d(nT)] \quad \text{in IC15, IC16}, \quad (7) \\
&= 0 \quad \text{in IC17, IC19}, \quad (8) \\
&= -L \quad \text{in IC18}, \quad (9) \\
&= L \quad \text{in IC20}. \quad (10)
\end{aligned}$$

To summarize, we have determined all the control rules and formulas for the fuzzy PD controller, with the control law (1) and the incremental control $\Delta u(nT)$ calculated by (2)–(10), according to the different locations in Fig. 3 of the error signal $e(nT)$ and the rate of the change of the error signal $r(nT)$. The initial conditions for the overall control system are the following natural values: for the incremental control, $\Delta u(0) = 0$; for the system output, $y(0) = 0$; and for the error and rate signals, $e(0) = sp$ (the set-point) and $r(0) = 0$.

Finally, we note that in the steady-state situation, $|e(nT)| = 0$, so that $|r(nT)| = |d(nT)| = 0$ in the denominators of the coefficients of $\Delta u(nT)$. Hence, we obtain the steady-state relations between the conventional PD control gains K_d^c and K_p^c and the fuzzy PD control gains K_p and K_d as follows:

$$K_d^c = \frac{K_u K_d}{4} \quad \text{and} \quad K_p^c = \frac{K_u K_p}{4}. \quad (11)$$

3. Simulation Results

This section shows some computer simulation results obtained by using the fuzzy PD controller for linear and nonlinear systems, with a comparison to the conventional PD controller. In the case that the system under control is linear first- or second-order, computer simulations results show that the fuzzy PD controller works as well as the conventional PD controller. It is not advantageous to apply the fuzzy PD controller for these cases. However, in the case that the system under control is linear but has time delay, and particularly for nonlinear systems, the fuzzy PD controller is significantly better than the conventional

PD controller, as shown in the simulation results described below.

In the following simulations, the x-axis is the discrete-time interval, T is the sampling period and n is the number of recursions shown on the x-axes of the figures.

We first consider a set of lower-order linear systems with time-delays that have been used for comparison. We observed that the fuzzy PD controller produces lower overshots and faster convergence in general, as can be seen from the example shown in Fig. 4. In this simulation, the system transfer function is

$$H(s) = \frac{1}{(100s+1)^2} e^{-2t}.$$

The conventional PD controller, with $K_p^c = 50.0$, $K_d^c = 20.0$, and $T = 0.1$, produces the solid-curve. On the other hand, the fuzzy PD controller with $T = 0.1, K_p = 51.8, K_d = 9.3, K_u = 0.6$, and $L = 16.0$ yields the dashed-curve in the same figure.

In the second case, the conventional and fuzzy PD controllers have been compared, using two nonlinear systems. The computer simulation results demonstrate that the fuzzy PD controller outperforms the conventional one significantly. The first one has the simple nonlinear model

$$\dot{y}(t) = 0.0001|y(t)| + u(t).$$

We used the constant set-point $sp = 1.0$ as the reference in this paper. Even for this constant set-point case, the conventional PD controller cannot handle this nonlinear system no matter how one changes its two constant gains. One control performance is shown in Fig. 5 (a), with $K_p^c = 3.0, K_d^c = 0.1$, and $T = 0.1$. In contrast, the fuzzy PD controller, with the parameters $T = 0.1, K_p = 19.5, K_d = 0.5, K_u = 0.1$, and $L = 20.0$, performed the tracking well, as shown in Fig. 5 (b).

The second example of nonlinear processes is shown in Fig. 6, where the nonlinear system is

$$\dot{y}(t) = -y(t) + y^2(t) + u(t).$$

Using the set-point $sp = 1.0$, we have compared the two PD controllers. The fuzzy controller is designed with $K_p = 35.0, K_d = 1.0, K_u = 0.1, L = 150.0$, and $T = 0.1$, which produces a very good tracking response as shown in Fig. 6 (the lower curve). However, no matter how one designs the two constant gains of the conventional PD controller, it does not show any reasonable results. In Fig. 6, the upper curve is one system output example of the conventional PD control system, with $K_p^c = 1.0, K_d^c = 0.01$, and $T = 0.1$.

4. Conclusions

Our many computer simulation results clearly indicate that the fuzzy PD controller works as well as the conventional PD controller for the first and second order linear systems [4]. Furthermore, the fuzzy PD controller has remarkable performance in the case of time-delay and nonlinear systems. It is worth mentioning that although still in its infancy, with many basic theories such as controllability and stability remaining to be further addressed, fuzzy control technology has been shown to be successful in many computer simulations and industrial applications, which make the investigation of various fuzzy control theories and techniques more challenging and interesting. As part of the ongoing research, we have investigated the stability of the closed-loop fuzzy PD control system in [3]. It is our belief that fuzzy control technology has a great potential in nontraditional systems control and hence deserves further investigation and development.

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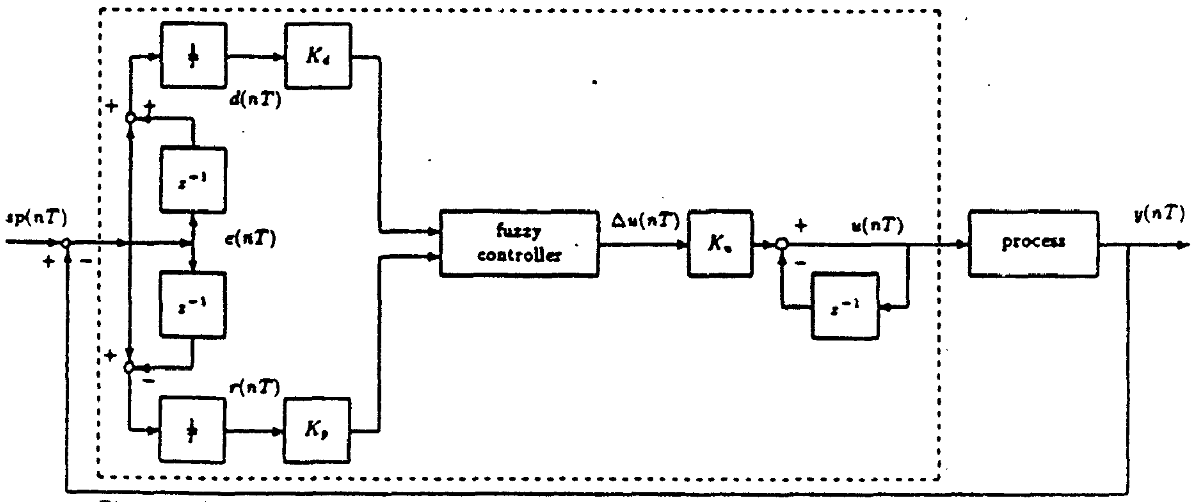

Figure 1. The closed-loop fuzzy PD control system.

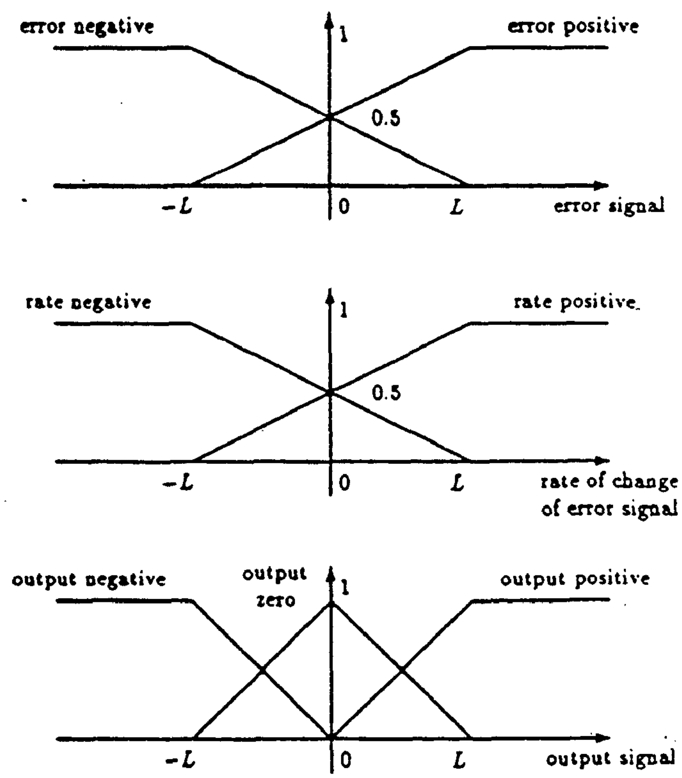

Figure 2. The membership functions of $e(nT)$, $r(nT)$ and $u(nT)$.

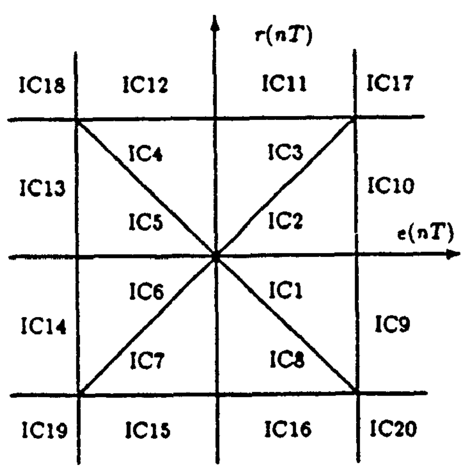

Figure 3. Regions of the fuzzy controller input-combination values.

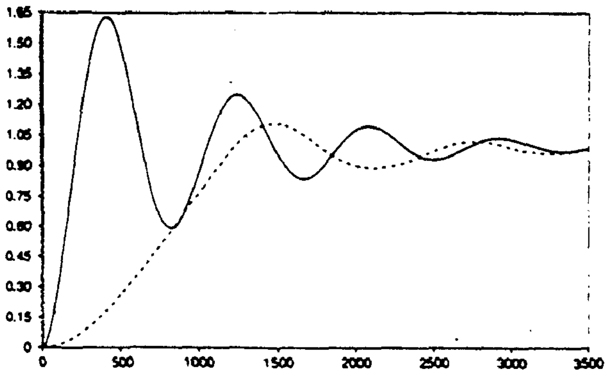

Figure 4. Time delay example.

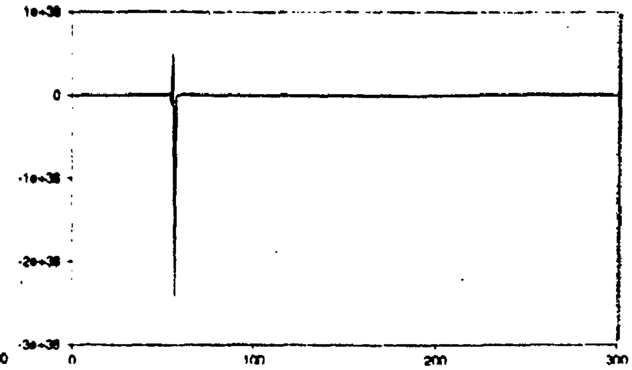

Figure 5a. Response of conventional PD for nonlinear system.

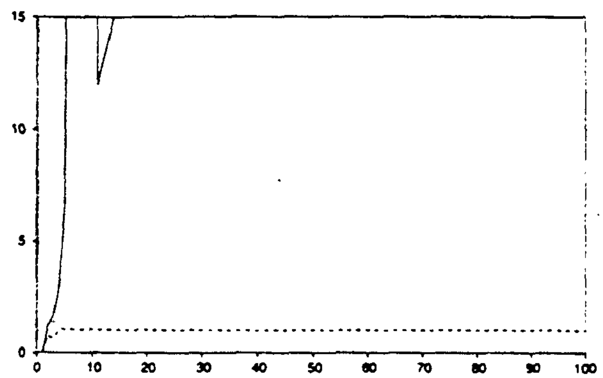

Figure 6. Responses of both conventional and fuzzy PD for nonlinear system.

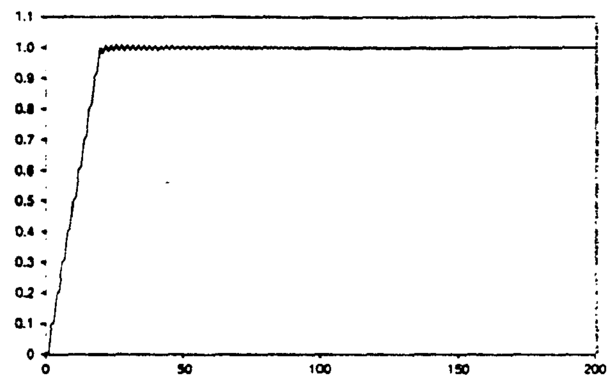

Figure 5b. Response of fuzzy PD for nonlinear system.

x-axis: # of sampling period, y-axis: reference.